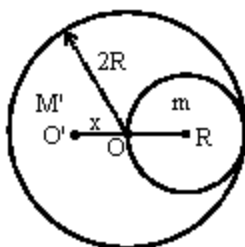


## HINTS & SOLUTIONS

### SECTION I - PHYSICS

1. (b) Let O be the centre of mass of the disc having radius  $2R$ .  $O'$  is the new C.M.



Let  $m$  = mass of disc of radius  $R$

$M'$  = mass of disc when the disc of radius  $R$  is removed

$M$  = mass of disc of radius  $2R$

Now,  $m = (\pi R^2)\sigma$ ,

where  $\sigma = \frac{M}{\pi(2R)^2} = \frac{M}{4\pi R^2}$  = the mass

per unit area

$$M' = [\pi(2R)^2 - \pi R^2]\sigma$$

$$= 3\pi R^2\sigma$$

$$M = \pi(2R)^2\sigma = 4\pi R^2\sigma$$

$$\text{We have, } \frac{M'x + mR}{M' + m} = 0$$

( $\because$  C.M. of the full disc is at the centre O)

$$\alpha, M'x + mR = 0$$

$$\alpha, M'x = -mR$$

$$\Rightarrow x = \left(-\frac{m}{M'}\right)R$$

$$= \left(-\frac{\pi R^2\sigma}{3\pi R^2\sigma}\right)R = \left(-\frac{1}{3}\right)R$$

$$\text{But } x = \frac{\alpha}{R}$$

$$\therefore \frac{\alpha}{R} = \left(-\frac{1}{3}\right)R$$

There appears misprint in this question.

There must be  $\alpha R$  instead of  $\frac{\alpha}{R}$ . Then

$$\alpha R = \left(-\frac{1}{3}\right)R \Rightarrow \alpha = -\frac{1}{3}$$

$$\therefore |\alpha| = \frac{1}{3}$$

2. (b) The acceleration of a solid sphere of mass  $M$ , radius  $R$  and moment of inertia  $I$  rolling down (without slipping) an inclined plane making an angle  $\theta$  with the horizontal is given by

$$a = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}}, \text{ where, } I = MK^2$$

3. (d) Central forces always act along the axis of rotation. Therefore, the torque is zero. And if there is no external torque acting on a rotating body then its angular momentum is constant.

4. (b) Let the spring be compressed by  $x$ .

Clearly, Initial K.E. of block = Potential energy of spring + work done against friction

$$\alpha, \frac{1}{2}mv^2 = \frac{1}{2}kx^2 + fx$$

$$\alpha, \frac{1}{2} \times 2 \times (4)^2 = \left(\frac{1}{2} \times 10000 \times x^2\right) + 15x$$

$$\alpha, 16 = 5000x^2 + 15x$$

$$\alpha, 5000x^2 + 15x - 16 = 0$$

$$\therefore x = \frac{-15 \pm \sqrt{(15)^2 - 4 \times 5000 \times (-16)}}{2 \times 5000}$$

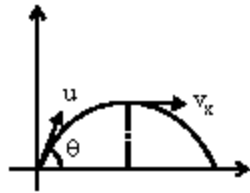
$$= \frac{-15 \pm 565.88}{10000}$$

(Ignoring -ve value)

$$\therefore x = 55 \text{ cm.}$$

5. (d) Let  $K'$  be the K.E. at the highest point. Then

$$K' = \frac{1}{2}mv_x^2 (\because v_y = 0 \text{ at highest point})$$



$$= \frac{1}{2} m (u \cos \theta)^2$$

$$= \frac{1}{2} m u^2 \cos^2 \theta = K \cos^2 \theta$$

$$\left( \because K = \frac{1}{2} m u^2 \right)$$

$$\alpha, K' = K \cos^2 60^\circ \quad (\because \theta = 60^\circ)$$

$$= K \left( \frac{1}{2} \right)^2 = \frac{K}{4}$$

6. (a) In young's double slit experiment, the intensity at a point is given by

$$I = I_0 \cos^2 \left( \frac{\phi}{2} \right)$$

where,  $I_0$  = maximum intensity

$\phi$  = phase difference

$$\text{Also, } \phi = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$$

$$\therefore I = I_0 \cos^2 \left( \frac{\pi}{6} \right)$$

$$\alpha, \frac{I}{I_0} = \cos^2 30^\circ = \left( \frac{\sqrt{3}}{2} \right)^2 = \frac{3}{4}$$

7. (a) The two springs are in parallel.  
 $\therefore$  Effective spring constant,  
 $K = K_1 + K_2$   
 Now, frequency of oscillation is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$$

$$\alpha, f = \frac{1}{2\pi} \sqrt{\frac{K_1 + K_2}{m}} \quad \dots \textcircled{1}$$

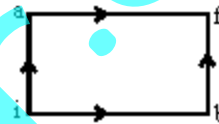
When both  $K_1$  and  $K_2$  are made four times their original values, the new frequency is given by

$$f' = \frac{1}{2\pi} \sqrt{\frac{4K_1 + 4K_2}{m}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{4(K_1 + K_2)}{m}} = 2 \left( \frac{1}{2\pi} \sqrt{\frac{K_1 + K_2}{m}} \right)$$

= 2f, from (1)

8. (b) For path iaf,  
 $Q = 50 \text{ cal}$   
 $W = 20 \text{ cal}$



By first law of thermodynamics,

$$\Delta U = Q - W = 50 - 20 = 30 \text{ cal.}$$

For path ibf

$$Q = 36 \text{ cal}$$

$$W = ?$$

By first law of thermodynamics,

$$Q = \Delta U + W$$

$$\alpha, W = Q - \Delta U$$

Since, the change in internal energy does not depend on the path, therefore

$$\Delta U = 30 \text{ cal}$$

$$\therefore W = Q - \Delta U = 36 - 30 = 6 \text{ cal.}$$

9. (b) The kinetic energy of a particle executing S.H.M. is given by

$$K = \frac{1}{2} m a^2 \omega^2 \sin^2 \omega t$$

where,  $m$  = mass of particle

$a$  = amplitude

$\omega$  = angular frequency

$t$  = time

Now, average K.E. =  $\langle K \rangle$

$$= \left\langle \frac{1}{2} m \omega^2 a^2 \sin^2 \omega t \right\rangle$$

$$= \frac{1}{2} m \omega^2 a^2 \langle \sin^2 \omega t \rangle$$

$$= \frac{1}{2} m \omega^2 a^2 \left( \frac{1}{2} \right) \quad \left( \because \langle \sin^2 \theta \rangle = \frac{1}{2} \right)$$

$$= \frac{1}{4} m \omega^2 a^2$$

$$= \frac{1}{4} m a^2 (2\pi\nu)^2 \quad (\because \omega = 2\pi\nu)$$

10. (b)  $\alpha_x < K > = \pi^2 m a^2 \nu^2$   
 Here,  $x = 2 \times 10^{-2} \cos \pi t$   
 Speed is given by

$$v = \frac{dx}{dt} = 2 \times 10^{-2} \pi \sin \pi t$$

For the first time, the speed to be maximum,  
 $\sin \pi t = 1$

$$\alpha_x \sin \pi t = \sin \frac{\pi}{2}$$

$$\Rightarrow \pi t = \frac{\pi}{2} \quad \alpha_x \quad t = \frac{1}{2} = 0.5 \text{ sec.}$$

11. (c) We know that power consumed in a.c. circuit is given by,  $P = E_{\text{rms}} I_{\text{rms}} \cos \phi$   
 Here,  $E = E_0 \sin \omega t$

$$I = I_0 \sin \left( \omega t - \frac{\pi}{2} \right)$$

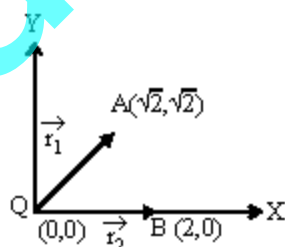
which implies that the phase difference,

$$\phi = \frac{\pi}{2}$$

$$\therefore P = E_{\text{rms}} I_{\text{rms}} \cos \frac{\pi}{2} = 0$$

$$\left( \because \cos \frac{\pi}{2} = 0 \right)$$

12. (c) The distance of point A( $\sqrt{2}, \sqrt{2}$ )



from the origin,

$$OA = |\vec{r}_1| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$$

$$= \sqrt{4} = 2 \text{ units.}$$

The distance of point B(2, 0) from the origin,

$$OB = |\vec{r}_2| = \sqrt{(2)^2 + (0)^2} = 2 \text{ units}$$

$$\text{Now, potential at A, } V_A = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{(OA)}$$

$$\text{Potential at B, } V_B = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{(OB)}$$

$\therefore$  Potential difference between the points A and B is given by

$$V_A - V_B = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{OA} - \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{OB}$$

$$= \frac{Q}{4\pi \epsilon_0} \left( \frac{1}{OA} - \frac{1}{OB} \right) = \frac{Q}{4\pi \epsilon_0} \left( \frac{1}{2} - \frac{1}{2} \right)$$

$$= \frac{Q}{4\pi \epsilon_0} \times 0 = 0.$$

13. (a) Required Ratio

$$= \frac{\text{Energy stored in capacitor}}{\text{Work done by the battery}}$$

$$= \frac{\frac{1}{2} C V^2}{C e^2}, \text{ where } C = \text{Capacitance of}$$

capacitor

$V =$  Potential difference,  
 $e =$  emf of battery

$$= \frac{\frac{1}{2} C e^2}{C e^2} \quad (\because V = e)$$

$$= \frac{1}{2}$$

14. (a) We have,  $I = I_0 \left( 1 - e^{-\frac{R}{L}t} \right)$

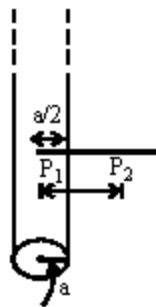
(When current is in growth in LR circuit)

$$= \frac{E}{R} \left( 1 - e^{-\frac{R}{L}t} \right) = \frac{5}{5} \left( 1 - e^{-\frac{5}{10} \times 2} \right)$$

$$= (1 - e^{-1})$$

15. (d) Here, current is uniformly distributed across the cross-section of the wire, therefore, current enclosed in the amperian path

formed at a distance  $r_1 \left( = \frac{a}{2} \right)$



$$= \left( \frac{\pi r_1^2}{\pi a^2} \right) \times I, \text{ where } I \text{ is total current}$$

$\therefore$  Magnetic field at

$$B_1(B_1) = \frac{\mu_0 \times \text{current enclosed}}{\text{Path}}$$

$$= \frac{\mu_0 \times \left( \frac{\pi r_1^2}{\pi a^2} \right) \times I}{2\pi r_1} = \frac{\mu_0 \times I r_1}{2\pi a^2}$$

Now, magnetic field at point  $P_2$ ,

$$(B_2) = \frac{\mu_0}{2\pi} \frac{I}{(2a)} = \frac{\mu_0 I}{4\pi a}$$

$$\therefore \text{ Required Ratio} = \frac{B_1}{B_2} = \frac{\mu_0 I r_1}{2\pi a^2} \times \frac{4\pi a}{\mu_0 I}$$

$$= \frac{2r_1}{a} = \frac{2 \times \frac{a}{2}}{a} = 1.$$

16. (d) There is no current inside the pipe and hence Ampere's law can not be applied.

17. (c) Binding energy

$$= [ZM_p + (A-Z)M_N - M]c^2$$

$$= [8M_p + (17-8)M_N - M]c^2$$

$$= [8M_p + 9M_N - M]c^2$$

$$= [8M_p + 9M_N - M_0]c^2$$

But the option (c) is negative of this.

18. (c) There is no change in the proton number and the neutron number as the  $\gamma$ -emission takes place as a result of excitation or de-excitation of nuclei.

19. (a) The current will flow through  $R_L$  when the diode is forward biased.

20. (a) Energy of a photon of frequency  $\nu$  is given by  $E = h\nu$ .

Also,  $E = pc$ , where  $p$  is the momentum of photon

$$\therefore h\nu = pc \Rightarrow p = \frac{h\nu}{c}$$

21. (c) We know that

$$v = \frac{dx}{dt} \Rightarrow dx = v dt$$

$$\text{Integrating, } \int_0^x dx = \int_0^t v dt$$

$$\text{or } x = \int_0^x (v_0 + gt + ft^2) dt$$

$$= \left[ v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3} \right]_0^t$$

$$\text{or } x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3} + c$$

where  $c$  is the constant of integration.

By question,

$$x = 0 \text{ at } t = 0.$$

$$\therefore 0 = v_0 \times 0 + \frac{g}{2} \times 0 + \frac{f}{3} \times 0 + c$$

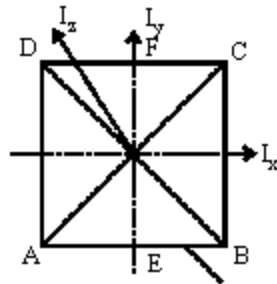
$$\Rightarrow c = 0.$$

$$\therefore x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$$

At  $t = 1$ ,

$$x = v_0 + \frac{g}{2} + \frac{f}{3}.$$

22. (d) By the theorem of perpendicular axes,  
 $I_z = I_x + I_y$        $\alpha$      $I_z = 2I_y$   
 ( $\therefore I_x = I_y$  by symmetry of the figure)



$$\therefore I_{EF} = \frac{I_z}{2}$$

... (i)

Again, by the same theorem

$$I_z = I_{AC} + I_{BD} = 2I_{AC}$$

( $\therefore I_{AC} = I_{BD}$  by symmetry of the figure)

$$\therefore I_{AC} = \frac{I_z}{2}$$

... (ii)

From (i) and (ii), we get

$$I_{EF} = I_{AC}$$

23. (a)

Here,

$$x = x_0 \cos(\omega t - \pi/4)$$

$\therefore$  Velocity

$$v = \frac{dx}{dt} = -x_0 \omega \sin\left(\omega t - \frac{\pi}{4}\right)$$

Acceleration,

$$a = \frac{dv}{dt} = -x_0 \omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$= x_0 \omega^2 \cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right] = x_0 \omega^2$$

$$\cos\left(\omega t + \frac{3\pi}{4}\right)$$

But by question,

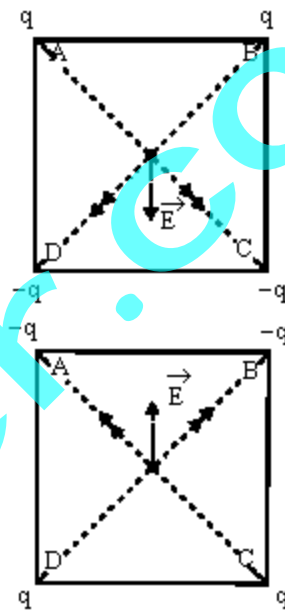
$$\text{Acceleration, } a = A \cos(\omega t + \delta)$$

Comparing the two accelerations, we get

$$A = x_0 \omega^2 \text{ and } \delta = \frac{3\pi}{4}$$

24. (a) As shown in the figure, the resultant electric fields before and after interchanging the charges will have the same magnitude but opposite directions.

Also, the potential will be same in both cases as it is a scalar quantity.



25. (b) By question,

Half life of X,  $T_{1/2} = \tau_{av}$ , average life of Y

$$\Rightarrow \frac{\ln 2}{\lambda_x} = \frac{1}{\lambda_y} \quad \alpha \quad \lambda_x = (\ln 2) \cdot \lambda_y$$

$$\Rightarrow \lambda_x = (0.693) \cdot \lambda_y$$

$$\therefore \lambda_x < \lambda_y$$

Now, the rate of decay is given by

$$R = R_0 e^{-\lambda t}$$

$$\text{For X, } R_x = R_0 e^{-\lambda_x t}$$

$$\text{For Y, } R_y = R_0 e^{-\lambda_y t}$$

Hence,  $R_x > R_y$ .

Thus, X will decay faster than Y.

26. (c) The efficiency ( $\eta$ ) of a Carnot engine and the coefficient of performance ( $\beta$ ) of a refrigerator are related as

$$\beta = \frac{1 - \eta}{\eta}$$

Here,  $\eta = \frac{1}{10}$

$$\therefore \beta = \frac{1 - \frac{1}{10}}{\left(\frac{1}{10}\right)} = 9.$$

Also, Coefficient of performance ( $\beta$ ) is given by  $\beta = \frac{Q_2}{w}$ , where  $Q_2$  is the energy absorbed from the reservoir.

$$\alpha, \quad 9 = \frac{Q_2}{10}$$

$$\therefore Q_2 = 90 \text{ J.}$$

27. (a) Si and Ge are semiconductors but C is an insulator. Also, the conductivity of Si and Ge is more than C because the valence electrons of Si, Ge and C lie in third, fourth and second orbit respectively.

28. (b) Here,  $\vec{E}$  and  $\vec{B}$  are perpendicular to each other and the velocity  $\vec{v}$  does not change, therefore

$$q\vec{E} = q\vec{v} \times \vec{B} \Rightarrow v = \frac{E}{B}$$

$$\text{Also, } \left| \frac{\vec{E} \times \vec{B}}{B^2} \right| = \frac{E B \sin\theta}{B^2}$$

$$= \frac{E B \sin 90^\circ}{B^2} = \frac{E}{B} = |\vec{v}| = v$$

$\therefore$  Option (b) is correct.

29. (a) Here,  $V(x) = \frac{20}{x^2 - 4}$  volt

$$\text{We know that } E = -\frac{dv}{dx} = -\frac{d}{dx} \left( \frac{20}{x^2 - 4} \right)$$

$$\alpha, \quad E = +\frac{40x}{(x^2 - 4)^2}$$

$$\text{At } x = 4 \mu\text{m},$$

$$E = +\frac{40 \times 4}{(4^2 - 4)^2} = +\frac{160}{144} = +\frac{10}{9} \text{ volt}/\mu\text{m}.$$

Positive sign indicates that  $\vec{E}$  is in +ve x-direction.

30. (d) We have to find the frequency of emitted photons. For emission of photons the transition must take place from a higher energy level to a lower energy level which are given only in options (c) and (d). Frequency is given by

$$h\nu = -13.6 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For transition from  $n=6$  to  $n=2$ ,

$$\nu_1 = \frac{-13.6}{h} \left( \frac{1}{6^2} - \frac{1}{2^2} \right) = \frac{2}{9} \times \left( \frac{13.6}{h} \right)$$

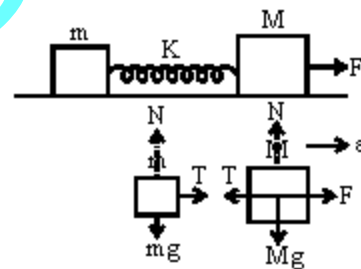
For transition from  $n=2$  to  $n=1$ ,

$$\nu_2 = \frac{-13.6}{h} \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = \frac{3}{4} \times \left( \frac{13.6}{h} \right).$$

$$\therefore \nu_1 > \nu_2$$

Hence option (d) is the correct answer.

31. (d) Writing free body diagrams for  $m$  &  $M$ ,



we get

$$T = ma \quad \text{and} \quad F - T = Ma$$

where  $T$  is force due to spring

$$\Rightarrow F - ma = Ma$$

$$\alpha, \quad F = Ma + ma$$

$$\therefore a = \frac{F}{M + m}$$

Now, force acting on the block of mass  $m$  is

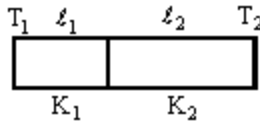
$$ma = m \left( \frac{F}{M + m} \right) = \frac{mF}{m + M}$$

32. (c) Power of combination is given by  $P = P_1 + P_2 = (-15 + 5) \text{ D} = -10 \text{ D}$ .

$$\text{Now, } P = \frac{1}{f} \Rightarrow f = \frac{1}{P} = \frac{1}{-10} \text{ metre}$$

$$\therefore f = -\left( \frac{1}{10} \times 100 \right) \text{ cm} = -10 \text{ cm}.$$

33. (d) Let  $T$  be the temperature of the interface. As the two sections are in series, the rate of flow of heat in them will be equal.



$$\therefore \frac{K_1 A (T_1 - T)}{l_1} = \frac{K_2 A (T - T_2)}{l_2}$$

where  $A$  is the area of cross-section.

$$\begin{aligned} \alpha \quad K_1 A (T_1 - T) l_2 &= K_2 A (T - T_2) l_1 \\ \alpha \quad K_1 T_1 l_2 - K_1 T l_2 &= K_2 T l_1 - K_2 T_2 l_1 \\ \alpha \quad (K_2 l_1 + K_1 l_2) T &= K_1 T_1 l_2 + K_2 T_2 l_1 \\ \therefore T &= \frac{K_1 T_1 l_2 + K_2 T_2 l_1}{K_2 l_1 + K_1 l_2} \\ &= \frac{K_1 l_2 T_1 + K_2 l_1 T_2}{K_1 l_2 + K_2 l_1} \end{aligned}$$

34. (a) We have,  $L_1 = 10 \log \left( \frac{I_1}{I_0} \right)$
- $$L_2 = 10 \log \left( \frac{I_2}{I_0} \right)$$
- $$\therefore L_1 - L_2 = 10 \log \left( \frac{I_1}{I_0} \right) - 10 \log \left( \frac{I_2}{I_0} \right)$$
- $$\alpha \quad \Delta L = 10 \log \left( \frac{I_1 \times I_0}{I_2 \times I_0} \right)$$
- $$\alpha \quad \Delta L = 10 \log \left( \frac{I_1}{I_2} \right)$$
- $$\alpha \quad 20 = 10 \log \left( \frac{I_1}{I_2} \right)$$
- $$\alpha \quad 2 = \log \left( \frac{I_1}{I_2} \right)$$
- $$\alpha \quad \frac{I_1}{I_2} = 10^2$$
- $$\alpha \quad I_2 = \frac{I_1}{100}$$
- $\Rightarrow$  Intensity decreases by a factor 100.

35. (b) We have,  
Molar heat capacity = Molar mass  $\times$  Specific heat

capacity per unit mass

$$\therefore C_p = 28 C_p \quad (\text{for nitrogen})$$

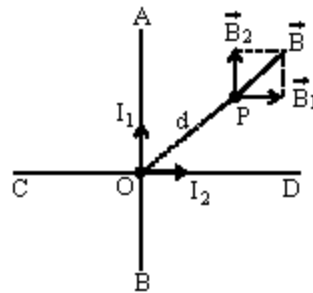
$$\text{and } C_v = 28 C_v$$

$$\text{Now, } C_p - C_v = R$$

$$\alpha \quad 28 C_p - 28 C_v = R$$

$$\Rightarrow C_p - C_v = \frac{R}{28}$$

36. (b) When a charged particle enters a magnetic field at a direction perpendicular to the direction of motion, the path of the motion is circular. In circular motion the direction of velocity changes at every point (the magnitude remains constant). Therefore, the momentum will change at every point. But kinetic energy will remain constant as it is given by  $\frac{1}{2} m v^2$  and  $v^2$  is the square of the magnitude of velocity which does not change.
37. (c) Clearly, the magnetic fields at a point  $P$ , equidistant from  $AOB$  and  $COD$  will have directions perpendicular to each other, as they are placed normal to each other.



$$\therefore \text{Resultant field } B = \sqrt{B_1^2 + B_2^2}$$

But  $B_1 = \frac{\mu_0 I_1}{2\pi d}$  and  $B_2 = \frac{\mu_0 I_2}{2\pi d}$

$$\therefore B = \sqrt{\left( \frac{\mu_0}{2\pi d} \right)^2 (I_1^2 + I_2^2)}$$

$$\alpha \quad B = \frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)^{1/2}$$

38. (d) We know that  
 $R_1 = R_0(1 + \alpha t)$   
 $\Rightarrow R_{50} = R_0(1 + 50\alpha)$  ... (i)  
 $R_{100} = R_0(1 + 100\alpha)$  ... (ii)  
 From (i),  $R_{50} - R_0 = 50\alpha R_0$  ... (iii)  
 From (ii),  $R_{100} - R_0 = 100\alpha R_0$  ... (iv)  
 Dividing (iii) by (iv), we get  

$$\frac{R_{50} - R_0}{R_{100} - R_0} = \frac{1}{2}$$
 Here,  $R_{50} = 5\Omega$  and  $R_{100} = 6\Omega$   
 $\therefore \frac{5 - R_0}{6 - R_0} = \frac{1}{2}$   
 $\alpha, 6 - R_0 = 10 - 2R_0$   
 $\alpha, R_0 = 4\Omega$ .

39. (a) The potential energy of a charged capacitor

$$\text{is given by } U = \frac{Q^2}{2C}$$

If a dielectric slab is inserted between the

plates, the energy is given by  $\frac{Q^2}{2KC}$ , where

K is the dielectric constant.

Again, when the dielectric slab is removed slowly its energy increases to initial potential energy. Thus, work done is zero.

40. (b) Electronic charge does not depend on acceleration due to gravity as it is a universal constant.

So, electronic charge on earth

= electronic charge on moon

$\therefore$  Required ratio = 1.

### SECTION II - CHEMISTRY

41. (b) According to Kohlrausch's law, molar conductivity of weak electrolyte acetic acid ( $\text{CH}_3\text{COOH}$ ) is given as follows:

$$\Lambda^\circ \text{CH}_3\text{COOH} = \Lambda^\circ \text{CH}_3\text{COONa} + \Lambda^\circ \text{HCl} - \Lambda^\circ \text{NaCl}$$

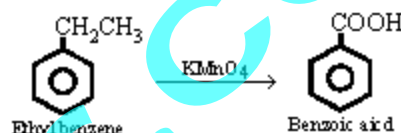
$\therefore$  Value of  $\Lambda^\circ_{\text{NaCl}}$  should also be

known for calculating value of  $\Lambda^\circ \text{CH}_3\text{COOH}$ .

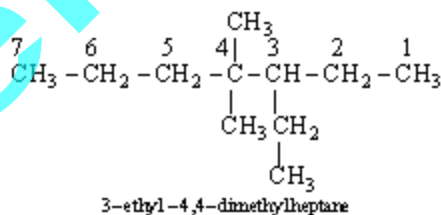
42. (d) Aromatic amines are less basic than aliphatic amines. Among aliphatic amines the order of basicity is  $2^\circ > 1^\circ > 3^\circ$  ( $\therefore$  of decreased electron density due to crowding in  $3^\circ$  amines)

$\therefore$  dimethylamine ( $2^\circ$  aliphatic amine) is strongest base among given choices.

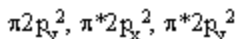
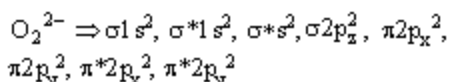
43. (d) When alkyl benzene are oxidised with alkaline  $\text{KMnO}_4$ , the entire alkyl group is oxidised to  $-\text{COOH}$  group regardless of length of side chain.



44. (a)



45. (b) Diamagnetic species have no unpaired electrons



46. (c) Reluctance of valence shell electrons to participate in bonding is called inert pair effect. The stability of lower oxidation state (+2 for group 14 element) increases on going down the group. So the correct order is  $\text{SiX}_2 < \text{GeX}_2 < \text{PbX}_2 < \text{SnX}_2$

47. (d) Chlorine reacts with excess of ammonia to produce ammonium chloride and nitrogen.



48. (d) Smaller the size and higher the charge more will be polarising power of cation. So the correct order of polarising power is  $\text{K}^+ < \text{Ca}^{2+} < \text{Mg}^{2+} < \text{Be}^{2+}$

49. (d) Mass of 3.6 moles of  $\text{H}_2\text{SO}_4$

$$= \text{Moles} \times \text{Molecular mass}$$

$$= 3.6 \times 98 \text{ g} = 352.8 \text{ g}$$

$$\therefore 1000 \text{ ml solution has } 352.8 \text{ g of } \text{H}_2\text{SO}_4$$

Given that 29 g of  $H_2SO_4$  is present in = 100 g of solution

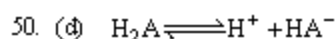
$\therefore$  352.8 g of  $H_2SO_4$  is present in

$$= \frac{100}{29} \times 352.8 \text{ g of solution}$$

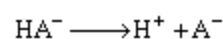
$$= 1216 \text{ g of solution}$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{1216}{1000} = 1.216 \text{ g/ml}$$

$$= 1.22 \text{ g/ml}$$



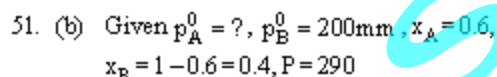
$$\therefore K_1 = 1.0 \times 10^{-5} = \frac{[H^+][HA^-]}{[H_2A]}$$



$$\therefore K_2 = 5.0 \times 10^{-10} = \frac{[H^+][A^-]}{[HA^-]}$$

$$K = \frac{[H^+]^2[A^{2-}]}{[H_2A]} = K_1 \times K_2$$

$$= (1.0 \times 10^{-5}) \times (5 \times 10^{-10}) = 5 \times 10^{-15}$$



$$P = p_A + p_B = p_A^0 x_A + p_B^0 x_B$$

$$\Rightarrow 290 = p_A^0 \times 0.6 + 200 \times 0.4$$

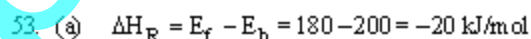
$$\therefore p_A^0 = 350 \text{ mm}$$



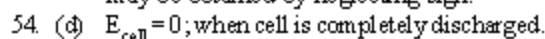
For a spontaneous reaction  $\Delta G^\circ < 0$

$$\text{or } \Delta H^\circ - T\Delta S^\circ < 0 \quad \Rightarrow \quad T > \frac{\Delta H^\circ}{\Delta S^\circ}$$

$$\Rightarrow T > \frac{179.3 \times 10^3}{160.2} > 1117.9 \text{ K} \approx 1118 \text{ K}$$



The nearest correct answer given in choices may be obtained by neglecting sign.



$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{0.059}{2} \log \left( \frac{[Zn^{2+}]}{[Cu^{2+}]} \right)$$

$$\text{or } 0 = 1.1 - \frac{0.059}{2} \log \left( \frac{[Zn^{2+}]}{[Cu^{2+}]} \right)$$

$$\log \left( \frac{[Zn^{2+}]}{[Cu^{2+}]} \right) = \frac{2 \times 1.1}{0.059} = 37.3$$

$$\therefore \left( \frac{[Zn^{2+}]}{[Cu^{2+}]} \right) = 10^{37.3}$$



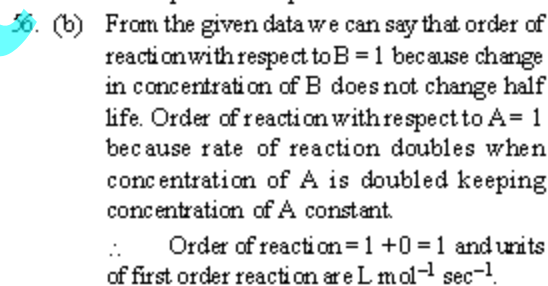
Given  $pK_a = 4.5$  and acid is 50% ionised.

$[HA] = [A^-]$  (when acid is 50% ionised)

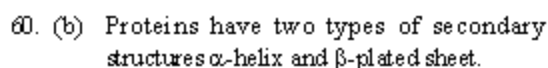
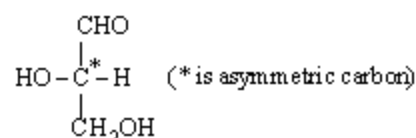
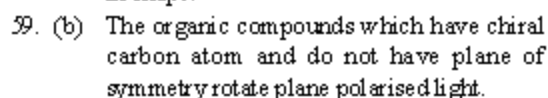
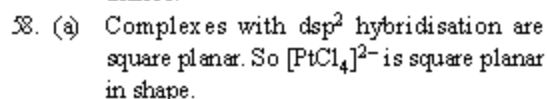
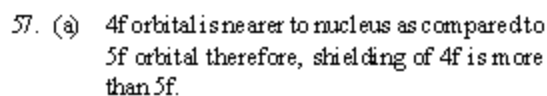
$$\therefore pH = pK_a + \log 1$$

$$\therefore pH = pK_a = 4.5$$

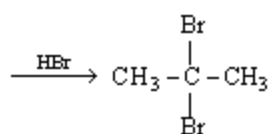
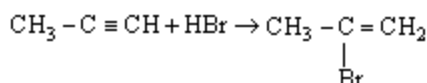
$$pOH = 14 - pH = 14 - 4.5 = 9.5$$



$\therefore$  Order of reaction = 1 + 0 = 1 and units of first order reaction are  $L \text{ mol}^{-1} \text{ sec}^{-1}$ .



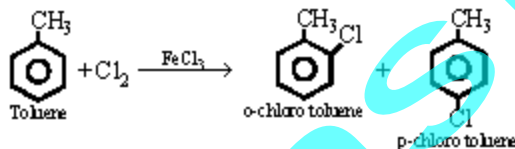
61. (b) The reaction follows Markownikoff rule which states that when unsymmetrical reagent adds across unsymmetrical double or triple bond the negative part adds to carbon atom having lesser number of hydrogen atoms.



2, 2-dibromo-propane

62. (a) This is carbylamine reaction.  
 $\text{CH}_3\text{CH}_2\text{NH}_2 + \text{CHCl}_3 + 3\text{KOH}$   
 $\longrightarrow \text{C}_2\text{H}_5\text{NC} + 3\text{KCl}$

63. (d)  $\text{FeCl}_3$  is Lewis acid. In presence of  $\text{FeCl}_3$  side chain hydrogen atoms of toluene are substituted.



64. (a) Nitro is electron withdrawing group, so it deactivates the ring towards electrophilic substitution.

65. (c)  
 (a)  $\text{N}_2$ : bond order 3, paramagnetic  
 $\text{N}_2^-$ : bond order 2.5, paramagnetic  
 (b)  $\text{C}_2$ : bond order 2, diamagnetic  
 $\text{C}_2^+$ : bond order 1.5, paramagnetic  
 (c)  $\text{NO}$ : bond order 2.5, paramagnetic  
 $\text{NO}^+$ : bond order 3, diamagnetic  
 (d)  $\text{O}_2$ : bond order 2, paramagnetic  
 $\text{O}_2^+$ : bond order 2.5, paramagnetic  
 $\therefore$  (c) is correct answer

66. (a) More the distance between nucleus and outer orbitals, lesser will be force of attraction on them. Distance between nucleus and 5f orbitals is more as compared to distance between 4f orbital and nucleus. So actinoids exhibit more number of oxidation states in general than the lanthanoids.

67. (d) Let the mass of methane and oxygen = m gm.  
 Mole fraction of  $\text{O}_2$

$$= \frac{\text{Moles of O}_2}{\text{Moles of O}_2 + \text{Moles of CH}_4}$$

$$= \frac{m/32}{m/32 + m/16} = \frac{m/32}{3m/32} = \frac{1}{3}$$

Partial pressure of  $\text{O}_2$  = Total pressure  $\times$  mole

$$\text{fraction of O}_2 = P \times \frac{1}{3} = \frac{1}{3}P$$

68. (a) Osmotic pressure of isotonic solutions ( $\pi$ ) are equal. For solution of unknown substance ( $\pi = CRT$ )

$$C_1 = \frac{5.25/M}{V}$$

For solution of urea,  $C_2$  (concentration)

$$= \frac{1.5/60}{V}$$

Given

$$\pi_1 = \pi_2$$

$$\therefore \pi = CRT$$

$$\therefore C_1RT = C_2RT \quad \text{or} \quad C_1 = C_2$$

$$\text{or} \quad \frac{5.25/M}{V} = \frac{1.5/60}{V}$$

$$\therefore M = 210 \text{ g/mol}$$

69. (d) Given  $\Delta H = 41 \text{ kJ mol}^{-1} = 41000 \text{ J mol}^{-1}$

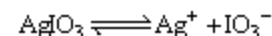
$$T = 100^\circ\text{C} = 273 + 100 = 373 \text{ K}$$

$$n = 1$$

$$\Delta U = \Delta H - \Delta nRT = 41000 - (2 \times 8.314 \times 373)$$

$$= 37898.88 \text{ J mol}^{-1} = 37.9 \text{ kJ mol}^{-1}$$

70. (c) Let x = solubility



$$K_{sp} = [\text{Ag}^+][\text{I}^-] = x \times x = x^2$$

$$\text{Given } K_{sp} = 1 \times 10^{-8}$$

$$\therefore x = \sqrt{K_{sp}} = \sqrt{1 \times 10^{-8}} = 1.0 \times 10^{-4}$$

mol/lit

$$= 1.0 \times 10^{-4} \times 283 \text{ g/lit}$$

$$= \frac{1.0 \times 10^{-4} \times 283 \times 100}{1000} \text{ gm/100 ml}$$

$$= 2.83 \times 10^{-3} \text{ gm/100 ml}$$

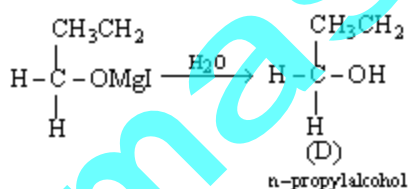
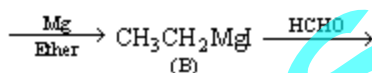
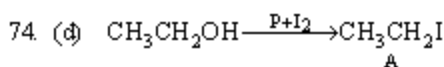
71. (a) Let activity of safe working = A  
Given  $A_0 = 10A$

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{30}$$

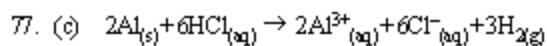
$$t_{90\%} = \frac{2.303}{\lambda} \log \frac{A_0}{A} = \frac{2.303}{0.693/30} \log \frac{10A}{A}$$

$$= \frac{2.303 \times 30}{0.693} \times \log 10 = 100 \text{ days.}$$

72. (b) Chiral conformation will not have plane of symmetry. Since twisted boat does not have plane of symmetry it is chiral.
73. (c) In  $S_N^2$  mechanism transition state is pentavalent. For bulky alkyl group it will have sterical hinderance and smaller alkyl group will favour the  $S_N^2$  mechanism. So the decreasing order of reactivity of alkyl halides is
- $$RCH_2X > R_2CHX > R_3CX$$



75. (c)
- (a)  $n=3, \ell=0$  means 3s-orbital  
(b)  $n=3, \ell=1$  means 3p-orbital  
(c)  $n=3, \ell=2$  means 3d-orbital  
(d)  $n=4, \ell=0$  means 4s-orbital
- Increasing order of energy among these orbitals is
- $$3s < 3p < 4s < 3d$$
- $\therefore$  3d has highest energy.
76. (c) Greater the difference between electronegativity of bonded atoms, stronger will be bond.
- $\therefore$  F-H ..... F is the strongest bond.

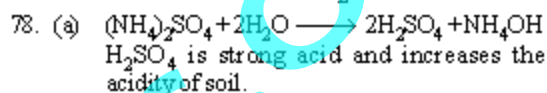


$\therefore$  6 moles of HCl produces = 3 moles of  $H_2$   
=  $3 \times 22.4$  L of  $H_2$

$\therefore$  1 mole of HCl produces  
=  $\frac{3 \times 22.4}{6}$  = 11.2 L of  $H_2$

$\therefore$  2 moles of Al produces 3 moles of  $H_2$   
=  $3 \times 22.4$  L of  $H_2$

$\therefore$  1 mole of Al produces  
=  $\frac{3 \times 22.4}{2}$  = 33.6 L of  $H_2$

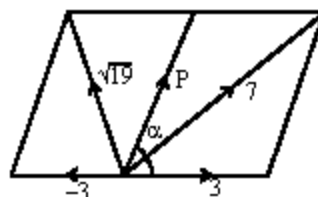


79. (b) Spontaneity of reaction depends on tendency to acquire minimum energy state and maximum randomness. For a spontaneous process in an isolated system the change in entropy is positive.

80. (b) Isotopes are atoms of same element having same atomic number but different atomic masses. Neutron has atomic number 0 and atomic mass 1. So loss of neutron will generate isotope.

### SECTION III - MATHEMATICS

81. (c) Given: Force  $P = Pn, Q = 3n$ , resultant  $R = 7n$   
&  $P' = Pn, Q' = (-3)n, R' = \sqrt{19}$



We know that

$$R^2 = P^2 + Q^2 + 2PQ \cos \alpha$$

$$\Rightarrow (7)^2 = P^2 + (3)^2 + 2 \times P \times 3 \cos \alpha$$

$$\Rightarrow 49 = P^2 + 9 + 6P \cos \alpha \quad \dots (i)$$

$$\Rightarrow 40 = P^2 + 6P \cos \alpha$$

$$\text{and } (\sqrt{19})^2 = P^2 + (-3)^2 + 2P \times -3 \cos \alpha$$

$$\Rightarrow 19 = P^2 + 9 - 6P \cos \alpha$$

$$\Rightarrow 10 = P^2 - 6P \cos \alpha \quad \dots (ii)$$

Adding (i) and (ii)

$$50 = 2P^2$$

$$\Rightarrow P^2 = 25 \Rightarrow P = 5n$$

82. (d) Given : Probabilities of aeroplane I, i.e.,  $P(I) = 0.3$  probabilities of scoring a target correctly by aeroplane II, i.e.  $P(II) = 0.2$   
 $\therefore P(\bar{I}) = 1 - 0.3 = 0.7$  and  $P(\bar{II}) = 1 - 0.2 = 0.8$   
 $\therefore$  The required probability  
 $= P(\bar{I} \cap \bar{II}) = P(\bar{I}) \cdot P(\bar{II}) = 0.7 \times 0.2 = 0.14$

83. (d) Given,  $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$   
 Apply  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$

$$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$$

Hence, D is divisible by both x and y

84. (b) Given, equation of hyperbola  
 $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$

We know that the equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Here,  $a^2 = \cos^2 \alpha$  and  $b^2 = \sin^2 \alpha$

We know that,  $b^2 = a^2(e^2 - 1)$

$$\begin{aligned} \Rightarrow \sin^2 \alpha &= \cos^2 \alpha (e^2 - 1) \\ \Rightarrow \sin^2 \alpha + \cos^2 \alpha &= \cos^2 \alpha \cdot e^2 \\ \Rightarrow e^2 &= 1 + \tan^2 \alpha = \sec^2 \alpha \Rightarrow e = \sec \alpha \end{aligned}$$

$$\therefore ae = \cos \alpha \cdot \frac{1}{\cos \alpha} = 1$$

Co-ordinates of foci are  $(\pm ae, 0)$  i.e.  $(\pm 1, 0)$   
 Hence, abscissae of foci remain constant when  $\alpha$  varies.

85. (b) Let the angle of line makes with the positive direction of z-axis is  $\alpha$  direction cosines of line with the +ve directions of x-axis, y-axis, and z-axis is l, m, n respectively.

$$\therefore l = \cos \frac{\pi}{4}, m = \cos \frac{\pi}{4}, n = \cos \alpha$$

as we know that,  $l^2 + m^2 + n^2 = 1$

$$\therefore \cos^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} + \cos^2 \alpha = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = 0 \Rightarrow \alpha = \frac{\pi}{2}$$

Hence, angle with positive direction of the

$$z\text{-axis is } \frac{\pi}{2}$$

86. (c) Using Lagrange's Mean Value Theorem  
 Let  $f(x)$  be a function defined on  $[a, b]$

$$\text{then, } f'(c) = \frac{f(b) - f(a)}{b - a} \quad \dots (i)$$

$c \in [a, b]$

$$\therefore \text{ Given } f(x) = \log_e x$$

$$\therefore f(x) = \frac{1}{x}$$

$\therefore$  equation (i) become

$$\frac{1}{c} = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \frac{1}{c} = \frac{\log_e 3 - \log_e 1}{2} = \frac{\log_e 3}{2}$$

$$\Rightarrow c = \frac{2}{\log_e 3} \Rightarrow c = 2 \log_3 e$$

87. (d) Given  $f(x) = \tan^{-1}(\sin x + \cos x)$

$$f(x) = \frac{1}{1 + (\sin x + \cos x)^2} (\cos x - \sin x)$$

$$= \frac{\sqrt{2} \left( \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x \right)}{1 + (\sin x + \cos x)^2}$$

$$= \frac{\left( \cos \frac{\pi}{4} \cdot \cos x - \sin \frac{\pi}{4} \cdot \sin x \right)}{1 + (\sin x + \cos x)^2}$$

$$\therefore f(x) = \frac{\sqrt{2} \cos \left( x + \frac{\pi}{4} \right)}{1 + (\sin x + \cos x)^2}$$

iff  $f'(x) > 0$  then  $f(x)$  is increasing function.

Hence  $f(x)$  is increasing, if  $-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$

$$\Rightarrow -\frac{3\pi}{4} < x < \frac{\pi}{4}$$

Hence,  $f(x)$  is increasing when  $n \in \left( -\frac{\pi}{2}, \frac{\pi}{4} \right)$

88. (a) Given  $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$  and  $|A^2| = 25$

$$\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$$

$$\therefore |\mathbf{A}^2| = 25 (25\alpha^2)$$

$$\therefore 25 = 25 (25\alpha^2) \Rightarrow |\alpha| = \frac{1}{5}$$

89. (d) We know that  $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$   
Put  $x = -1$

$$\therefore e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots$$

$$\therefore e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots$$

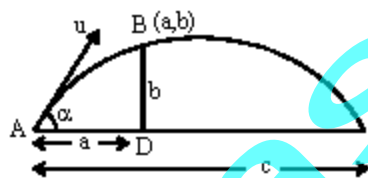
90. (b) Given  $|\hat{u} \times 3\hat{v}| = 1$  and  $\theta$  is acute angle between  $\hat{u}$  and  $\hat{v}$ ,  $|\hat{u}| = 1, |\hat{v}| = 1$

$$\Rightarrow 6|\hat{u}||\hat{v}|\sin\theta = 1$$

$$\Rightarrow 6|\sin\theta| = 1 \Rightarrow \sin\theta = \frac{1}{6}$$

Hence, there is exactly one value of  $\theta$  for which  $2\hat{u} \times 3\hat{v}$  is a unit vector.

91. (a) Let B be the top of the wall whose coordinates will be (a, b). Range (R) = c



B lies on the trajectory

$$y = x \tan\alpha - \frac{1}{2}g \frac{x^2}{u^2 \cos^2\alpha}$$

$$\Rightarrow b = a \tan\alpha - \frac{1}{2}g \frac{a^2}{u^2 \cos^2\alpha}$$

$$\Rightarrow b = a \tan\alpha \left[ 1 - \frac{ga}{2u^2 \cos^2\alpha \tan\alpha} \right]$$

$$= a \tan\alpha \left[ 1 - \frac{a}{\frac{2u^2}{g} \cos^2\alpha \frac{\sin\alpha}{\cos\alpha}} \right]$$

$$= a \tan\alpha \left[ 1 - \frac{a}{\frac{u^2 \cdot 2 \sin\alpha \cos\alpha}{g}} \right]$$

$$= a \tan\alpha \left[ 1 - \frac{a}{\frac{u^2 \sin 2\alpha}{g}} \right]$$

$$= a \tan\alpha \left[ 1 - \frac{a}{R} \right] \quad \left( \because R = \frac{u^2 \sin^2\alpha}{g} \right)$$

$$\Rightarrow b = a \tan\alpha \left[ 1 - \frac{a}{c} \right]$$

$$\Rightarrow b = a \tan\alpha \left( \frac{c-a}{c} \right)$$

$$\Rightarrow \tan\alpha = \frac{bc}{a(c-a)}$$

The angle of projection,  $\alpha = \tan^{-1} \frac{bc}{a(c-a)}$

92. (a) Let the number of boys be x and that of girls be y.

$$\Rightarrow 52x + 42y = 50(x+y)$$

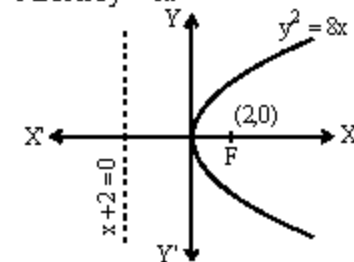
$$\Rightarrow 52x - 50x = 50y - 42y$$

$$\Rightarrow 2x = 8y \Rightarrow \frac{x}{y} = \frac{4}{1} \text{ and } \frac{x}{x+y} = \frac{4}{5}$$

$$\text{Required \% of boys} = \frac{x}{x+y} \times 100$$

$$= \frac{4}{5} \times 100 = 80\%$$

93. (b) Parabola  $y^2 = 8x$



Point must be on the directrix of parabola  
 $\therefore$  equation of directrix  $x+2=0 \Rightarrow x=-2$   
Hence the point is  $(-2, 0)$

94. (c) We know that equation of sphere is  $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

where centre is  $(-u, -v, -w)$

$$\text{given } x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$$

$$\therefore \text{centre} \equiv (3, 6, 1)$$

Coordinates of one end of diameter of the sphere are  $(2, 3, 5)$ . Let the coordinates of the other end of diameter are  $(\alpha, \beta, \gamma)$

$$\therefore \frac{\alpha+2}{2} = 3, \frac{\beta+3}{2} = 6, \frac{\gamma+5}{2} = 1$$

$$\Rightarrow \alpha = 4, \beta = 9 \text{ and } \gamma = -3$$

$\therefore$  Coordinate of other end of diameter are  $(4, 9, -3)$

95. (b) Given  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$  and

$$\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$$

If  $\vec{c}$  lies in the plane of  $\vec{a}$  and  $\vec{b}$ , then  $[\vec{a} \ \vec{b} \ \vec{c}] = 0$

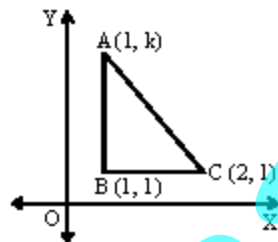
$$\text{i.e. } \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & (x-2) & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1[1-2(x-2)] - 1[-1-2x] + 1[x-2+x] = 0$$

$$\Rightarrow 1-2x+4+1+2x+2x-2=0$$

$$\Rightarrow 2x = -4 \Rightarrow x = -2$$

96. (a) Given: The vertices of a right angled triangle  $A(1, k), B(1, 1)$  and  $C(2, 1)$  and Area of  $\Delta ABC = 1$  square unit

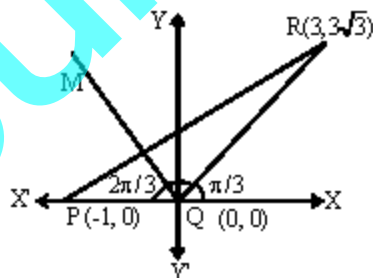


We know that, area of right angled triangle

$$= \frac{1}{2} \times BC \times AB = 1 = \frac{1}{2}(1)(k-1)$$

$$\Rightarrow \pm(k-1) = 2 \Rightarrow k = -1, 3$$

97. (c) Given: The coordinates of points P, Q, R are  $(-1, 0), (0, 0), (3, 3\sqrt{3})$  respectively.



$$\text{Slope of QR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3\sqrt{3}}{3}$$

$$\Rightarrow \tan \theta = \sqrt{3}$$

$$\Rightarrow \theta = \frac{\pi}{3} \Rightarrow \angle RQX = \frac{\pi}{3}$$

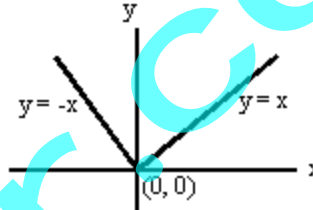
$$\therefore \angle RQC = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\therefore \text{Slope of the line QM} = \tan \frac{2\pi}{3} = -\sqrt{3}$$

$$\therefore \text{Equation of line QM is } (y-0) = -\sqrt{3}(x-0)$$

$$\Rightarrow y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$$

98. (a) Equation of bisectors of lines,  $xy=0$  are  $y = \pm x$



$\therefore$  Put  $y = \pm x$  in the given equation on

$$my^2 + (1-m^2)xy - mx^2 = 0$$

$$\therefore mx^2 + (1-m^2)x^2 - mx^2 = 0$$

$$\Rightarrow 1-m^2=0 \Rightarrow m = \pm 1$$

99. (c) Given  $f(x) = f(x) + f\left(\frac{1}{x}\right)$ , where  $f(x) = \int_1^x \frac{\log t}{1+t} dt$

$$\therefore F(e) = f(e) + f\left(\frac{1}{e}\right)$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt \dots (A)$$

$$\text{Now for solving } I = \int_1^{1/e} \frac{\log t}{1+t} dt$$

$$\therefore \text{Put } \frac{1}{t} = z \Rightarrow -\frac{1}{t^2} dt = dz \Rightarrow dt = -\frac{dz}{z^2}$$

$$\text{and limit for } t=1 \Rightarrow z=1 \text{ and for } t=1/e \Rightarrow z=e$$

$$\therefore I = \int_1^e \frac{\log\left(\frac{1}{z}\right)}{1+\frac{1}{z}} \left(-\frac{dz}{z^2}\right)$$

$$= \int_1^e \frac{(\log 1 - \log z)z}{z+1} \left(-\frac{dz}{z^2}\right)$$

$$= \int_1^e -\frac{\log z}{(z+1)} \left(-\frac{dz}{z}\right) \quad [\because \log 1 = 0]$$

$$= \int_1^e \frac{\log z}{z(z+1)} dz$$

$$\therefore I = \int_1^e \frac{\log t}{t(t+1)} dt$$

[By property  $\int_a^b f(t)dt = \int_a^b f(x)dx$  ]

Equation(A) be

$$F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$$

$$= \int_1^e \frac{t \cdot \log t + \log t}{t(1+t)} dt = \int_1^e \frac{(\log t)(t+1)}{t(1+t)} dt$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{t} dt$$

Let  $\log t = x$

$$\therefore \frac{1}{t} dt = dx$$

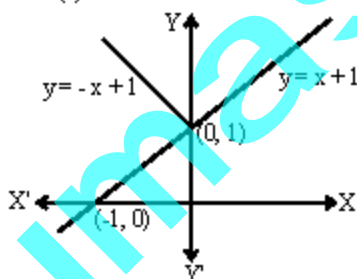
[for limit  $t=1, x=0$  and  $t=e, x=\log e=1$ ]

$$\therefore F(e) = \int_0^1 x dx$$

$$F(e) = \left[ \frac{x^2}{2} \right]_0^1$$

$$\Rightarrow F(e) = \frac{1}{2}$$

100. (a)  $f(x) = \min\{x+1, |x|+1\}$   
 $\Rightarrow f(x) = x+1 \quad \forall x \in \mathbb{R}$



Hence,  $f(x)$  is differentiable everywhere for all  $x \in \mathbb{R}$ .

101. (b) Given,  $f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$

$$\Rightarrow f(0) = \lim_{x \rightarrow 0} \frac{1}{x} - \frac{2}{e^{2x}-1}$$

$$= \lim_{x \rightarrow 0} \frac{(e^{2x}-1) - 2x}{x(e^{2x}-1)} \quad \left[ \frac{0}{0} \text{ form} \right]$$

$\therefore$  using L'Hospital rule

$$f(0) = \lim_{x \rightarrow 0} \frac{4e^{2x}}{2(xe^{2x} + e^{2x} \cdot 1) + e^{2x} \cdot 2}$$

$$= \lim_{x \rightarrow 0} \frac{4e^{2x}}{4xe^{2x} + 2e^{2x} + 2e^{2x}} \quad \left[ \frac{0}{0} \text{ form} \right]$$

$$= \lim_{x \rightarrow 0} \frac{4e^{2x}}{4(xe^{2x} + e^{2x})} = \frac{4e^0}{4(0 + e^0)} = 1$$

102. (c)  $\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$

$$\therefore \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x$$

$$\therefore \left[ \sec^{-1} t \right]_{\sqrt{2}}^x = \frac{\pi}{2}$$

$$\Rightarrow \sec^{-1} x - \sec^{-1} \sqrt{2} = \frac{\pi}{2}$$

$$\Rightarrow \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \sec^{-1} x = \frac{\pi}{2} + \frac{\pi}{4}$$

$$\Rightarrow \sec^{-1} x = \frac{3\pi}{4} \Rightarrow x = \sec \frac{3\pi}{4}$$

$$\Rightarrow x = -\sqrt{2}$$

103. (c)  $I = \int \frac{dx}{\cos x + \sqrt{3} \sin x}$

$$I = \int \frac{dx}{2 \left[ \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right]}$$

$$= \frac{1}{2} \int \frac{dx}{\left[ \sin \frac{\pi}{6} \cos x + \cos \frac{\pi}{6} \sin x \right]}$$

$$= \frac{1}{2} \int \frac{dx}{\sin \left( x + \frac{\pi}{6} \right)}$$

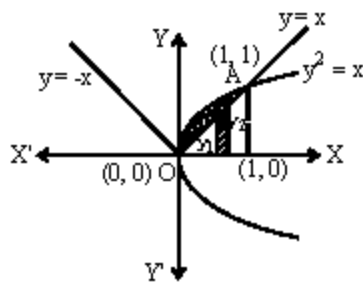
$$\Rightarrow I = \frac{1}{2} \int \operatorname{cosec} \left( x + \frac{\pi}{6} \right) dx$$

But we know that

$$\int \operatorname{cosec} x dx = \log |(\tan x / 2)| + C$$

$$\therefore I = \frac{1}{2} \cdot \log \tan \left( \frac{x}{2} + \frac{\pi}{2} \right) + C$$

104. (a) The area enclosed between the curves  $y^2 = x$  and  $y = |x|$   
From the figure, area lies between  $y^2 = x$  and  $y = x$



$$\begin{aligned} \therefore \text{Required area} &= \int_0^1 (y_2 - y_1) dx \\ &= \int_0^1 (\sqrt{x} - x) dx = \left[ \frac{x^{3/2}}{3/2} - \frac{x^2}{2} \right]_0^1 \\ \therefore \text{Required area} &= \frac{2}{3} \left[ x^{3/2} \right]_0^1 - \frac{1}{2} \left[ x^2 \right]_0^1 \\ &= \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \end{aligned}$$

105. (c) Let  $\alpha$  and  $\beta$  are roots of the equation  $x^2 + ax + 1 = 0$

$$\alpha + \beta = -a \text{ and } \alpha\beta = 1$$

$$\text{given } |\alpha - \beta| < \sqrt{5}$$

$$\Rightarrow \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} < \sqrt{5}$$

$$\left( \because (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta \right)$$

$$\begin{aligned} \Rightarrow \sqrt{a^2 - 4} < \sqrt{5} &\Rightarrow a^2 - 4 < 5 \\ \Rightarrow a^2 - 9 < 0 &\Rightarrow a^2 < 9 \Rightarrow -3 < a < 3 \\ \Rightarrow a \in (-3, 3) \end{aligned}$$

106. (b) Let the series  $a, ar, ar^2, \dots$  are in geometric progression.

$$\text{given, } a = ar + ar^2$$

$$\Rightarrow 1 = r + r^2$$

$$\Rightarrow r^2 + r - 1 = 0$$

$$\Rightarrow r = \frac{-1 \pm \sqrt{1 - 4 \times -1}}{2}$$

$$\Rightarrow r = \frac{-1 \pm \sqrt{5}}{2} \text{ (taking +ve value)}$$

$$\Rightarrow r = \frac{\sqrt{5} - 1}{2}$$

107. (d)  $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \operatorname{cosec}^{-1}\left(\frac{5}{4}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$$

$$[\because \sin^{-1} x + \cos^{-1} x = \pi/2]$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right) \dots (1)$$

$$\text{Let } \cos^{-1}\frac{4}{5} = A \Rightarrow \cos A = \frac{4}{5}$$

$$\Rightarrow A = \cos^{-1}(4/5)$$

$$\Rightarrow \sin A = \frac{3}{5}$$

$$\Rightarrow A = \sin^{-1}\frac{3}{5}$$

$$\therefore \cos^{-1}(4/5) = \sin^{-1}(3/5)$$

$$\therefore \text{equation (1) become,}$$

$$\sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$$

$$\Rightarrow \frac{x}{5} = \frac{3}{5} \Rightarrow x = 3$$

108. (c)  $T_{r+1} = (-1)^r \cdot {}^n C_r (a)^{n-r} \cdot (b)^r$  is an expansion of  $(a-b)^n$

$$\therefore \text{5th term} = t_5 = t_{4+1}$$

$$= (-1)^4 \cdot {}^n C_4 (a)^{n-4} (b)^4 = {}^n C_4 \cdot a^{n-4} \cdot b^4$$

$$\text{6th term} = t_6 = t_{5+1} = (-1)^5 \cdot {}^n C_5 (a)^{n-5} (b)^5$$

$$\text{Given } t_5 + t_6 = 0$$

$$\therefore {}^n C_4 \cdot a^{n-4} \cdot b^4 + (-1) \cdot {}^n C_5 \cdot a^{n-5} \cdot b^5 = 0$$

$$\Rightarrow \frac{n!}{4!(n-4)!} \cdot \frac{a^n}{a^4} b^4 - \frac{n!}{5!(n-5)!} \cdot \frac{a^n b^5}{a^5} = 0$$

$$\Rightarrow \frac{n! a^n b^4}{4!(n-4)! a^4} \left[ \frac{1}{(n-4)} - \frac{6}{5a} \right] = 0$$

$$\text{or, } \frac{1}{n-4} - \frac{6}{5a} = 0 \Rightarrow \frac{a}{b} = \frac{x-4}{5}$$

109. (a) Set  $S = \{1, 2, 3, \dots, 12\}$

$$A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \phi$$

$\therefore$  The number of ways to partition

$$= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4 = \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times \frac{4!}{4!0!}$$

$$= \frac{12!}{(4!)^3}$$

110. (b)  $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2}-1\right) + \log(\cos x)$

$f(x)$  is defined if  $-1 \leq \left(\frac{x}{2}-1\right) \leq 1$  and  $\cos x > 0$

or  $0 \leq \frac{x}{2} \leq 2$  and  $-\frac{\pi}{2} < x < \frac{\pi}{2}$

or  $0 \leq x \leq 4$  and  $-\frac{\pi}{2} < x < \frac{\pi}{2}$

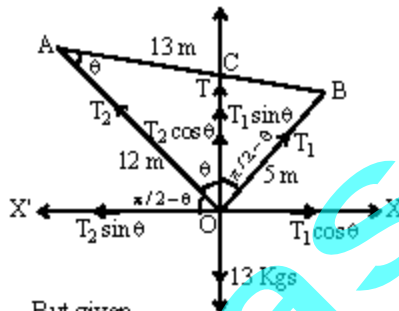
$\therefore x \in \left[0, \frac{\pi}{2}\right)$

111. (a) Given: A body weighing 13 kgs is suspended by two strings OB = 5m and OA = 12 m. Length of rod AB = 13 M.

Let  $T_1$  is tension in string OB and  $T_2$  is tension in string OA.

$\therefore T_2 \sin \theta = T_1 \cos \theta$  ... (i)

and  $T_1 \sin \theta + T_2 \cos \theta = 13$  ... (ii)



But given

$OC = CA = CB$

$\therefore \angle AOC = \angle OAC = \theta$  (let)

and  $\angle COB = \angle OBC$

Now in  $\Delta AOB$

$\sin \theta = \sin A = \frac{5}{13}$  and  $\cos \theta = \frac{12}{13}$

Now putting the value of  $\sin \theta$  and  $\cos \theta$  in equation (i) and (ii) we get

$T_2 \frac{5}{13} = T_1 \frac{12}{13}$  and  $T_1 \frac{5}{13} + T_2 \frac{12}{13} = 13$

$\Rightarrow 12T_1 - 5T_2 = 0$  ... (iii)

$\Rightarrow 5T_1 + 12T_2 = 169$  ... (iv)

Solving equation (iii) and (iv)

$60T_1 - 25T_2 = 0$

$-60T_1 + 144T_2 = -169 \times 12$

$-169T_2 = -169 \times 12$

$\Rightarrow T_2 = 12$  and  $T_1 = 5$

$\therefore$  Tensions in strings are 5kg and 12 kg

112. (b) A pair of fair dice is thrown, the sample space  $S = (1, 1), (1, 2), (1, 3), \dots = 36$

Possibility of getting 9 are (3, 4), (4, 5), (6, 3), (3, 6)

$\therefore$  Possibility of getting score 9 in a single throw

$= \frac{4}{36} = \frac{1}{9}$

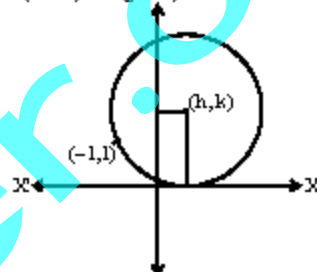
$\therefore$  Probability of getting score 9 exactly twice

$= {}^3C_2 \times \left(\frac{1}{9}\right)^2 \left(1 - \frac{1}{9}\right) = \frac{3!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9}$

$= \frac{3 \cdot 2!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9} = \frac{8}{243}$

113. (d) Equation of circle whose centre is (h, k)

i.e.  $(x-h)^2 + (y-k)^2 = k^2$



(radius of circle = k because circle is tangent to x-axis)

Equation of circle passing through (-1, 1)

$\therefore (-1-h)^2 + (1-k)^2 = k^2$

$\Rightarrow 1+h^2+2h+1+k^2-2k=k^2$

$\Rightarrow h^2+2h-2k+2=0$

$D \geq 0$

$\therefore (2)^2 - 4 \times 1 \times (-2k+2) \geq 0$

$\Rightarrow 4 - 4(-2k+2) \geq 0 \Rightarrow 1+2k-2 \geq 0$

$\Rightarrow k \geq \frac{1}{2}$

114. (c) Let the direction cosines of line L be l, m, n, then

$2l+3m+n=0$  ... (i)

and  $l+3m+2n=0$  ... (ii)

on solving equation (i) and (ii), we get

$\frac{l}{6-3} = \frac{m}{1-4} = \frac{n}{6-3} \Rightarrow \frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$

Now  $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{3^2+(-3)^2+3^2}}$

$\therefore l^2+m^2+n^2=1$

$\therefore \frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{1}{\sqrt{27}}$

$\Rightarrow l = \frac{3}{\sqrt{27}} = \frac{1}{\sqrt{3}}, m = -\frac{1}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}$

Line L, makes an angle  $\alpha$  with +ve x-axis

$\therefore l = \cos \alpha$

$\Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$

115. (a) General equation of circles passing through origin and having their centres on the x-axis  
 $x^2 + y^2 + 2gx = 0$  ... (i)  
 On differentiating w.r.t. x, we get

$$2x + 2y \cdot \frac{dy}{dx} + 2g = 0 \Rightarrow g = -\left(x + y \frac{dy}{dx}\right)$$

$\therefore$  equation (i) be

$$x^2 + y^2 + 2\left[-\left(x + y \frac{dy}{dx}\right)\right] \cdot x = 0$$

$$\Rightarrow x^2 + y^2 - 2x^2 - 2x \frac{dy}{dx} \cdot y = 0$$

$$\Rightarrow y^2 = x^2 + 2xy \frac{dy}{dx}$$

116. (c) Since, p and q are positive real numbers  
 $p^2 + q^2 = 1$  (Given)  
 Using AM  $\geq$  GM

$$\therefore \left(\frac{p+q}{2}\right)^2 \geq \sqrt{(pq)^2} = \frac{p^2 + q^2 + 2pq}{4} \geq pq$$

$$\frac{1+2pq}{4} \geq pq$$

$$1+2pq \geq 4pq$$

$$1 \geq 2pq$$

$$\alpha, 2pq \leq 1$$

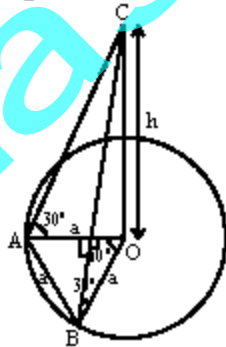
$$pq \leq \frac{1}{2}$$

$$\alpha, pq \leq \frac{1}{2}$$

$$\text{Now, } (p+q)^2 = p^2 + q^2 + 2pq$$

$$\Rightarrow (p+q)^2 \leq 1 + 2 \times \frac{1}{2} \Rightarrow p+q \leq \sqrt{2}$$

117. (a) In the  $\Delta AOB$ ,  
 $\angle AOB = 60^\circ$ ,  
 and  $\angle OBA =$   
 $\angle OAB$  (since  $OA = OB = AB$  radius of same circle).  $\therefore \Delta AOB$  is an equilateral triangle. Let the height of tower be h. Given distance between two points A & B lie on boundary of circular park, subtends an angle of  $60^\circ$  at the foot of the tower AB i.e.  $AB = a$ . A tower OC stands at the centre of a circular park. Angle of elevation of the top of the tower from A and B is  $30^\circ$ . In  $\Delta OAX$



$$\tan 30^\circ = \frac{h}{a}$$

$$\therefore \angle OBA = \angle AOB = \angle OAB = 60^\circ$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a} \Rightarrow h = \frac{a}{\sqrt{3}}$$

118. (d) We know that,  $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + \dots + {}^{20}C_{10} x^{10} + \dots + {}^{20}C_{20} x^{20}$   
 Put  $x = -1$ ,  $(0) = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots + {}^{20}C_{10} - {}^{20}C_{11} + \dots + {}^{20}C_{20}$   
 $\Rightarrow 0 = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots - {}^{20}C_9 + {}^{20}C_{10}]$   
 $\Rightarrow {}^{20}C_{10} = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots - {}^{20}C_9 + {}^{20}C_{10}]$   
 $\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots + {}^{20}C_{10}$   
 $= \frac{1}{2} {}^{20}C_{10}$

119. (b,c) Equation of normal at  $p(x, y)$  is

$$Y - y = -\frac{dx}{dy}(X - x)$$

Coordinate of G at X axis is  $(X, 0)$  (let)

$$\therefore 0 - y = -\frac{dx}{dy}(X - x)$$

$$\Rightarrow y \frac{dy}{dx} = X - x \Rightarrow X = x + y \frac{dy}{dx}$$

$\therefore$  Co-ordinate of G  $\left(x + y \frac{dy}{dx}, 0\right)$

Given distance of G from origin = twice of the abscissa of p.

$$\therefore \left|x + y \frac{dy}{dx}\right| = |2x|$$

$$\Rightarrow x + y \frac{dy}{dx} = 2x \quad \text{or} \quad x + y \frac{dy}{dx} = -2x$$

$$\Rightarrow y \frac{dy}{dx} = x \quad \text{or} \quad y \frac{dy}{dx} = -3x$$

$$\Rightarrow y dy = x dx \quad \text{or} \quad y dy = -3x dx$$

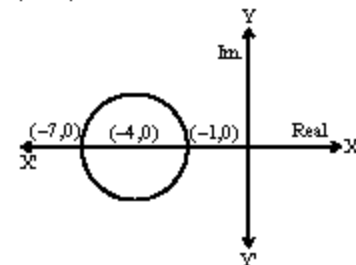
On Integrating

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c_1 \quad \text{or} \quad \frac{y^2}{2} = -\frac{3x^2}{2} + c_2$$

$$\Rightarrow x^2 - y^2 = -2c_1 \quad \text{or} \quad 3x^2 + y^2 = 2c_2$$

$\therefore$  the curve is a hyperbola and ellipse both z lies on or inside the circle with centre  $(-4, 0)$  and radius 3 units.

120. (a)



From the Argand diagram maximum value of  $|z+1| = 6$

$$\text{Second method : } |z+1| = |z+4-3|$$

$$\leq |z+4| + |-3| \leq |3| + |-3|$$

$$\Rightarrow |z+1| = 6$$