AIEEE Entrance Test - 2005

PHYSICS SOLUTIONS

1. Out of the following pair, which one does not have identical dimensions is

(1) moment of inertia and moment of a force (2) work and torque

(3) angular momentum and Planck's constant (4) impulse and momentum

Sol: Ans [1]

$\tau = I\alpha$ $\Rightarrow$ moment of force and moment of inertia will not have same dimensions.

2. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is

(1) $-2av^3$ (2) $2av^2$ (3) $-2abv^2$ (4) $2bv^3$

Sol: Ans [1]

$$\frac{dt}{dx} = 2ax + b \quad \Rightarrow \quad \frac{dx}{dt} = \frac{1}{(2ax+b)}$$

$$\frac{d^2x}{dt^2} = -(2ax+b)^{-2} \cdot 2a\frac{dx}{dt} = -2av^3$$

3. A car starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then

(1) $S = \frac{1}{2}ft^2$ (2) $S = \frac{1}{2}ft^2$ (3) $S = \frac{1}{4}ft$ (4) $S = \frac{1}{6}ft^2$

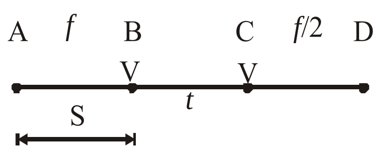
Sol: Ans []

$S = \frac{V^2}{2f}$ (i)

$S_2 = BC = V \cdot t$ (ii)

$CD = S_3 = \frac{V^2}{f} = 2S$ (iii)

$\Rightarrow$ $S_2 = 15\text{ S} - [AB + CD] = 12\text{ S}$ (iv)



Solving (i) - (iv), $S = \frac{f}{72}t^2$

There is no alternative matching

4. A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 seconds the velocity changes to

5 ms^{-1} northwards. The average acceleration in this time is

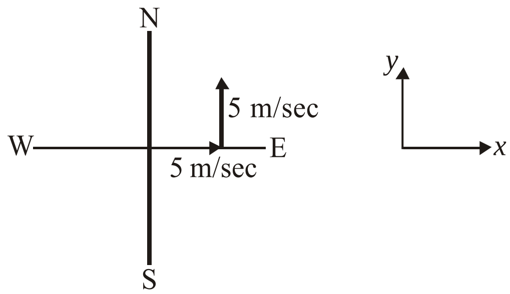
(1) zero

(2) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-west

(3) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-east

(4) $\frac{1}{2}$ ms^{-2} towards north

Sol: Ans [2]



$$\mathbf{a}_{avg} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t} = \frac{5\mathbf{j} - 5\mathbf{i}}{10} = \frac{1}{2}(-\mathbf{i} + \mathbf{j})$$

Average acceleration will be $\frac{1}{\sqrt{2}}$ m/sec^2 in

North-West direction

5. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the times of flights in the two cases, then the product of the two time of flights is proportional to

(1) $\frac{1}{R}$ (2) R (3) R^2 (4) $\frac{1}{R^2}$

Sol: Ans [2]

At complementary angle of projection i.e., $\theta_1 + \theta_2 = 90°$, ranges are same.

$$t_1 = \frac{2u \sin \theta_1}{g}; \; t_2 = \frac{2u \sin \theta_2}{g} = \frac{2u \sin(90 - \theta_1)}{g} = \frac{2u \cos \theta_1}{g}$$

$$t_1 t_2 = \frac{4u^2 \sin \theta_1 \cos \theta_1}{g} = \frac{2\,R}{g}$$

6. An angular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, $\frac{F_1}{F_2}$ is

(1) 1 (2) $\frac{R_1}{R_2}$ (3) $\frac{R_2}{R_1}$ (4) $\left(\frac{R_1}{R_2}\right)^2$

Sol: Ans [2]

There will be only centrifugal acceleration of all particles because angular speed is constant

$F_1 = ma_A = mR_1 w^2$

$F_2 = ma_B = mR_2 w^2$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{R_1}{R_2}$$

(Masses of particles are not mentioned in the question. This answer is subject to mass of particles is same)

7. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is

(1) $\mu_s = 1 - \frac{1}{n^2}$ (2) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$ (3) $\mu_k = 1 - \frac{1}{n^2}$ (4) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

Sol: Ans [3]

Let length of incline is d

For smooth inclined plane $d = 0 + \frac{1}{2} \times g \sin 45° \, t_1^2$ (i)

For rough inclined plane $d = 0 + \frac{1}{2}(g \sin 45° - \mu_k g \cos 45°) \cdot t_2^2$ (ii)

Using (i) & (ii), $\frac{1}{2} g \sin 45° \, t_1^2 = \frac{1}{2} g(\sin 45° - \mu_k \cos 45°) t_2^2$

$$\frac{1}{\sqrt{2}} t_1^2 = \frac{1}{\sqrt{2}} (1 - \mu_k) \cdot n t_1^2$$

$$\Rightarrow \quad (1 - \mu_k) = \frac{1}{n^2}$$

$$\Rightarrow \quad \mu_k = \left(1 - \frac{1}{n^2}\right)$$

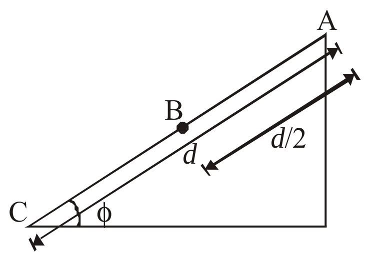
8. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by

(1) $2 \tan \phi$ (2) $\tan \phi$ (3) $2 \sin \phi$ (4) $2 \cos \phi$

Sol: Ans [1]

Let AB is smooth, BS is rough

Applying work energy theorem between A and C



$KE_i = 0, KE_f = 0 \quad \Rightarrow \Delta KE = 0$

$W_{mg} = mgd \cdot \sin\phi$

$W_{friction} = -(\mu mg \cos \phi) \times \frac{d}{2}$

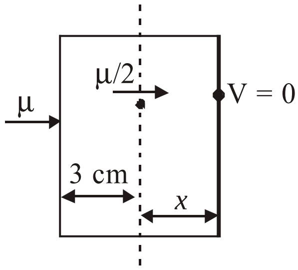
$\Rightarrow \quad mgd\sin\phi - \mu mg\cos\phi\,\dfrac{d}{2} = 0,\ \ 2\tan\phi = \mu$

9. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?

(1) 1.5 cm (2) 1.0 cm (3) 3.0 cm (4) 2.0 cm

Sol: Ans [2]

Using work energy theorem



$$\frac{1}{2}m\left(\frac{u}{2}\right)^2 - \frac{1}{2}mu^2 = -F\times 3$$

or $\dfrac{3}{8}mu^2 = F \times 3$ (i)

Also

$$\frac{1}{2}m(0)^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = -Fx \quad \text{or} \quad \frac{mu^2}{8} = -Fx \qquad \text{(ii)}$$

Using equation (i) & (ii)

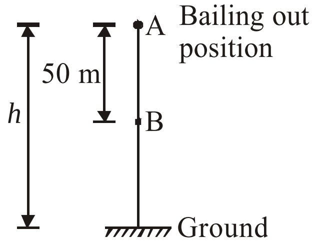
$$\frac{3}{1} = \frac{3}{x} \quad \Rightarrow \quad x = 1\text{ cm}$$

10. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. he reaches the ground with a speed of 3 m/s. At what height, did he bail out?

(1) 293 m (2) 111 m (3) 91 m (4) 182 m

Sol: Ans [1]

Applying work energy theorem



$W_{net} = mg \times 50 - m \times 2 \times (h - 50)$

$\Delta KE = \dfrac{1}{2}m(2)^2 - 0$

$\Rightarrow \quad mg \times 50 - m \times 2(h - 50) = \dfrac{1}{2}m(2)^2 \Rightarrow h = 293\ m$

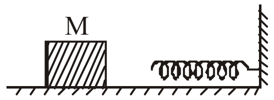
11. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point 20 cm away from the origin?

(1) 5 m/s^2 (2) 10 m/s^2 (3) 3 m/s^2 (4) 15 m/s^2

Sol: Ans [2]

At $x = 20\text{cm},\ a = \dfrac{15\times 0.2}{0.3} = 10\text{m/sec}^2$

12. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant K and compresses it by length L. The maximum momentum of the block after collision is

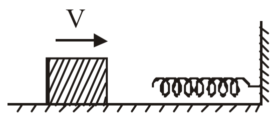


(1) zero (2) $\frac{ML^2}{K}$

(3) $\sqrt{MK}\ L$ (4) $\frac{KL^2}{2\ M}$

Sol: Ans [3]

Using conservation of energy



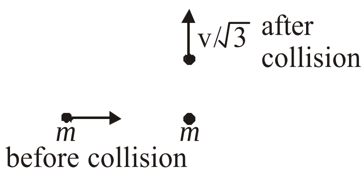
$$\frac{1}{2}mv^2 = \frac{1}{2}Kx^2$$

$$\Rightarrow \quad \frac{(m\,v)^2}{2m} = \frac{1}{2}K\,x^2$$

Maximum momentum

$$mv = \sqrt{KM}\ L \quad \text{(given } x = L)$$

13. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision



(1) $\frac{2}{\sqrt{3}}v$ (2) $\frac{v}{\sqrt{3}}$

(3) v (4) $\sqrt{3}\ v$

Sol: Ans [1]

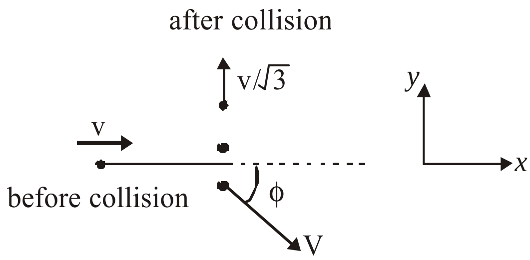
Since there is no external force acting therefore momentum will be conserved in all direction

Conserving momentum along x-axis :

$$mv = mV\cos\phi$$

$$\Rightarrow \quad V\cos\phi = v \qquad \text{(i)}$$

Conserving momentum along y-axis :



$$m\frac{v}{\sqrt{3}} = mV\sin\phi$$

$$\Rightarrow \quad V\sin\phi = \frac{v}{\sqrt{3}} \qquad \text{(ii)}$$

From (i) & (ii)

$$V^2 = \frac{4v^2}{3} \quad \Rightarrow \quad V = \frac{2v}{\sqrt{3}}$$

14. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is

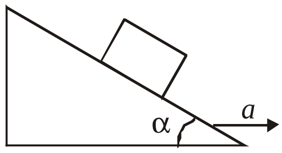
(1) 10 m/s (2) $10\sqrt{30}$ m/s (3) 40 m/s (4) 20 m/s

Sol: Ans [3]

Applying conservation of mechanical energy

$$\frac{1}{2}mv^2 = mg \times 80, \; v = 40 \text{ m/sec}$$

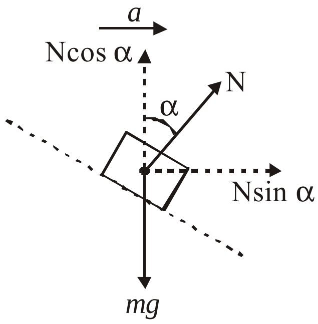
15. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then a is equal to



(1) g (2) $g \tan\alpha$

(3) $g / \tan\alpha$ (4) $g \operatorname{cosec}\alpha$

Sol: Ans [2]



Net acceleration of block is in horizontal direction

$N \cos\alpha = mg$

$N \sin\alpha = ma$

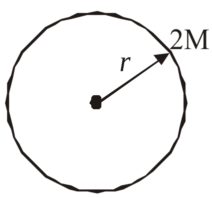
$\Rightarrow \tan\alpha = a/g$

$\Rightarrow a = g \tan\alpha$

16. The moment of inertia of a uniform semicircular disc of mass M and radius r about a lime perpendicular to the plane of the disc through the centre is

(1) Mr^2 (2) $\frac{1}{2}Mr^2$ (3) $\frac{1}{4}Mr^2$ (4) $\frac{2}{5}Mr^2$

Sol: Ans [2]



$$I = \frac{\frac{1}{2}(2M)r^2}{2} = \frac{1}{2}M r^2$$

17. A body A of mass M while falling vertically downwards under gravity breaks into two parts: a body B of mass $\frac{1}{3}$M and a body C of mass $\frac{2}{3}$M. The centre of mass of bodies B and C taken together shifts compared to that of body A towards

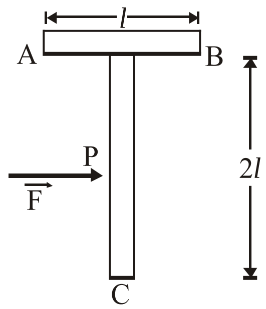
(1) body C (2) body B

(3) depends on height of breaking (4) does not shift

Sol: Ans [4]

When body A breaks into B & C there is no change in external forces and internal forces play no role for motion of centre of mass.

18. A T shaped object with dimensions shown in the figure, is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.



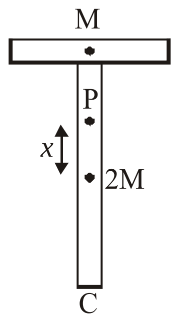
(1) $\frac{4}{3}l$ (2) l

(3) $\frac{2}{3}l$ (4) $\frac{3}{2}l$

Sol: Ans [1]

To have pure translatory motion, F must act at the centre of mass of the body.

Distance of centre of mass from mid-point O.



$$x = \frac{M}{3M} \cdot l = \frac{l}{3}$$

$\Rightarrow$ distance of P from C

$$= l + \frac{l}{3} = \frac{4l}{3}$$

19. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which one of the following is correct?

(1) $d = 2h$ (2) $d = h$ (3) $d = \frac{h}{2}$ (4) $d = \frac{3h}{2}$

Sol: Ans [1]

$$g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

$\Rightarrow$ $d = 2h$

20. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere (you may take $G = 6.67 \times 10^{-11}$ Nm^2/kg^2)

(1) 6.67×10^{-9} J (2) 6.67×10^{-10} J (3) 13.34×10^{-10} J (4) 3.33×10^{-10} J

Sol: Ans [2]

Work done = change in potential energy

$$= 0 - \left[\frac{G \times 100 \times 10 \times 10^{-3}}{0.1}\right]$$

$$= 6.67 \times 10^{-10} \text{ J}$$

21. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be

(1) 4 cm (2) 20 cm (3) 8 cm (4) 10 cm

Sol: Ans [3]

In freely falling elevator effect of gravity will disappear and also angle of contact will become 90°

22. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is

(1) $\frac{2Y}{S^2}$ (2) $\frac{S}{2Y}$ (3) $2S^2Y$ (4) $\frac{S^2}{2Y}$

Sol: Ans [4]

Formula

23. Average density of the earth

(1) is directly proportional to g

(2) is inversely proportionally to g

(3) does not depend on g

(4) is a complex function of g

Sol: Ans [1]

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} \times \frac{G}{G} = \frac{3g}{4\pi GR}$$

24. A body of mass m is accelerated uniform from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by

(1) $\frac{1}{2}\frac{mv^2}{T^2} \cdot t$ (2) $\frac{1}{2}\frac{mv^2}{T^2} \cdot t^2$ (3) $\frac{mv^2}{T^2} \cdot t$ (4) $\frac{mv^2}{T^2} \cdot t^2$

Sol: Ans [3]

$$v = 0 + aT \Rightarrow \quad a = \left(\frac{v}{T}\right)$$

Velocity at any time $t = \left(\frac{v}{T} \cdot t\right)$

$$P = F \cdot v = \frac{m\,v}{T} \cdot \frac{v}{T} \cdot t = \frac{m\,v^2}{T^2} \cdot t$$

25. Consider a car moving on a straight road with a speed of 100 m/s. The distance at which car can be stopped is [$\mu_k = 0.5$]

(1) 100 m (2) 400 m (3) 800 m (4) 1000 m

Sol: Ans [4]

$$\text{Maximum retardation} = -\frac{\mu_k mg}{m} = -\mu_k g$$

$$0 = (100)^2 - 2 \times \mu_k \times 10 \times S$$

$$S = 1000 \text{ m}$$

26. Which of the following is incorrect regarding the first law of thermodynamics?

(1) It introduces the concept of the internal energy

(2) It introduces the concept of the entropy

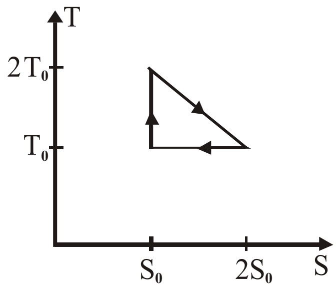
(3) It is not applicable to any cyclic process

(4) It is a restatement of the principle of conservation of energy

Sol: Ans [2]

Second law of thermodynamics introduces the concept of entropy

27. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is



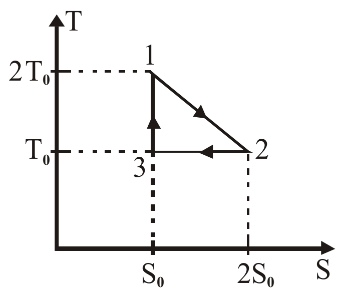
(1) $\frac{1}{3}$ (2) $\frac{2}{3}$

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Sol: Ans [1]

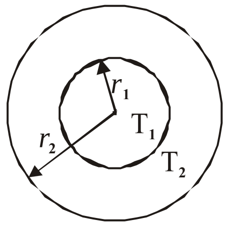
$$\text{Heat taken from source} = \text{Area under curve } 1 - 2 = \frac{3}{2} S_0 T_0$$

$$\text{Work done} = \text{Area of closed path} = \frac{1}{2} S_0 T_0$$



$$\eta = \frac{\frac{1}{2} S_0 T_0}{\frac{3}{2} S_0 T_0} = \frac{1}{3}$$

28. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperature T_1 and T_2 respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



(1) $\frac{r_1 r_2}{(r_2 - r_1)}$ (2) $(r_2 - r_1)$

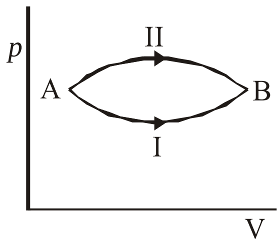
(3) $\frac{(r_2 - r_1)}{r_1 r_2}$ (4) $\ln \frac{r_2}{r_1}$

Sol: Ans [1]

$$\frac{dQ}{dt} = \frac{4\pi K(T_1 - T_2)}{(r_2 - r_1)} \cdot r_1 r_2$$

$$\Rightarrow \frac{dQ}{dt} \propto \frac{r_1 r_2}{(r_2 - r_1)}$$

29. A system goes from A to B via two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the process I and II respectively, then



(1) $\Delta U_2 > \Delta U_1$

(2) $\Delta U_2 < \Delta U_1$

(3) $\Delta U_1 = \Delta U_2$

(4) relation between ΔU_1 and ΔU_2 can not be determined

Sol: Ans [3]

Change in internal energy is independent of path

30. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio $\frac{C_p}{c_v}$ of the mixture is

(1) 1.4 (2) 1.54 (3) 1.59 (4) 1.62

Sol: Ans [4]

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

where r is for mixture

$$n_1 = \frac{16}{4} = 4; \; n_2 = \frac{16}{32} = 0.5$$

$$\Rightarrow \frac{4 + \frac{1}{2}}{\gamma - 1} = \frac{4}{\frac{5}{3} - 1} + \frac{\frac{1}{2}}{\frac{7}{5} - 1}; \; \gamma = 1.62$$

31. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $\frac{I}{8}$. The thickness of lead which will reduce the intensity to $\frac{I}{2}$ will be

(1) 18 mm (2) 12 mm (3) 6 mm (4) 9 mm

Sol: Ans [2]

$$I' = Ie^{-\mu x}$$

where μ is coefficient of penetration

$$\frac{I}{8} = Ie^{-\mu \times 36}$$

$$\frac{I}{2} = Ie^{-\mu \cdot x}$$

Solving $x = 12$ mm

32. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap in (eV) for the semiconductor is

(1) 0.5 eV (2) 0.7 eV (3) 1.1 eV (4) 2.5 eV

Sol: Ans [1]

$$\text{Band gap} = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{2580\times10^{-9}\times1.6\times10^{-19}} = 0.5 \text{ eV}$$

33. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed $\frac{1}{2}$ m away, the number of electrons emitted by photocathode would

(1) decrease by a factor of 2 (2) increase by a factor of 2
(3) decrease by a factor of 4 (4) increase by a factor of 4

Sol: Ans [4]

No of electrons emitted $(n) \propto \frac{1}{d^2}$

$$\Rightarrow \frac{n_1}{n_2} = \frac{d_2^{\,2}}{d_1^{\,2}}$$

$$n_2 = n_1 \frac{d_1^{\,2}}{d_2^{\,2}} = n_1 \times \frac{1^2}{(1/2)^2} = 4n_1$$

34. Starting with a sample of pure ^{66}Cu, $\frac{7}{8}$ of it decays into Zn in 15 minutes. The corresponding half-life is

(1) 5 minutes (2) $7\frac{1}{2}$ minutes (3) 10 minutes (4) 15 minutes

Sol: Ans [1]

Applying

$$N = N_0 e^{-\lambda t}$$

$$\Rightarrow \frac{1}{8}N_0 = N_0\, e^{-\lambda \times 15}$$

$$\Rightarrow \lambda \times 15 = \log_e 8$$

$$\lambda = \frac{3 \times 0.693}{15}$$

$$T_{1/2} = \frac{0.693}{\lambda} = 5\text{ min}$$

35. If radius of the $^{27}_{13}\text{Al}$ nucleus is estimated to be 3.6 Fermi then the radius of $^{125}_{52}\text{Te}$ nucleus be nearly

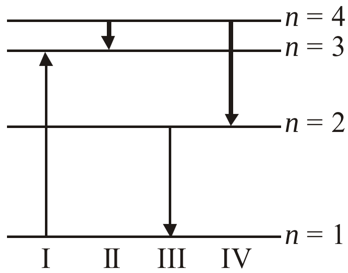
(1) 4 fermi (2) 5 fermi (3) 6 fermi (4) 8 fermi

Sol: Ans [3]

$\because$ $A \propto r^3$

$$\Rightarrow \frac{3.6}{r_{Te}} = \left(\frac{27}{125}\right)^{\frac{1}{3}} = \frac{3}{5} \Rightarrow r_{Te} = 6\text{ fermi}$$

36. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



(1) I (2) II

(3) III (4) IV

Sol: Ans [3]

Maximum energy gap is for transition from $n = 2 \rightarrow 1$

37. If the kinetic energy of a free electron doubles, its deBroglie wavelength changes by the factor

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$ (3) $\frac{1}{2}$ (4) 2

Sol: Ans [1]

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}} \; ; \; \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}}$$

$$\Rightarrow \; \lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

38. In a common base amplifier, the phase difference between the input signal voltage and output voltage is

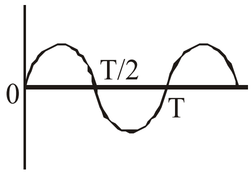
(1) 0 (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) π

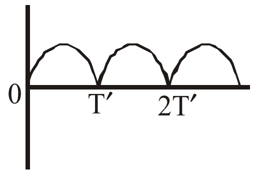
Sol: Ans [1]

39. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be

(1) 100 Hz (2) 70.7 Hz (3) 50 Hz (4) 25 Hz

Sol: Ans [1]





In full wave rectification frequency doubles

40. A nuclear transformation is denoted by $X(n, \alpha)\,^7_3\mathrm{Li}$. Which of the following is the nucleus of element X?

(1) $^9_5\mathrm{B}$ (2) $^{11}_4\mathrm{Be}$ (3) $^{12}\mathrm{C}_6$ (4) $^{10}_5\mathrm{B}$

Sol: Ans [4]

$$^{10}_5\mathrm{X} + {}^1_0n = {}^7_3\mathrm{Li} + {}^4_2\mathrm{He}$$

41. The function $\sin^2(\omega t)$ represents

(1) a simple harmonic motion with a period $2\pi/\omega$

(2) a simple harmonic motion with a period π/ω

(3) a periodic, but not simple harmonic motion with a period $2\pi/\omega$

(4) a periodic, but not simple harmonic motion with a period π/ω

Sol: Ans [2]

$$y = \sin^2(\omega t)$$

$$y = \frac{1}{2}(1 - \cos 2\omega t)$$

it represents S.H.M. with angular frequency 2ω

$$\Rightarrow \quad T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

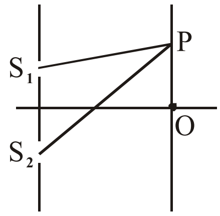
42. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is

(1) straight line (2) parabola (3) hyperbola (4) circle

Sol: Ans [3]

$$S_2P - S_1P = n\lambda$$

for a given n, difference of the distance of point P from two fixed points is constant therefore locus of any frinze will be hyperbola



43. Two simple harmonic motions are represented by the equation $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

(1) $-\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $-\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Sol: Ans [3]

Velocity leads displacement by $\pi/2$

$$\Rightarrow \quad V_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right); \; V_2 = 0.1 \sin\left(\pi t + \frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$\Delta\phi_{12} = \left(\frac{\pi}{3} + \frac{\pi}{2}\right) - \left(\frac{\pi}{2} + \frac{\pi}{2}\right) = -\frac{\pi}{6}$$

44. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the surface, the radius of this circle in cm is

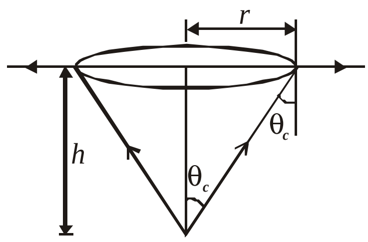
(1) $36\sqrt{5}$ (2) $4\sqrt{5}$ (3) $36\sqrt{7}$ (4) $36/\sqrt{7}$

Sol: Ans [4]

$\tan\theta_c = \frac{r}{h}$

$\Rightarrow \quad r = h\tan q_c$

$\because \quad \sin\theta_c = \frac{1}{\mu}$



$\tan\theta_c = \frac{1}{\sqrt{\mu^2 - 1}}$

$\Rightarrow \quad r = \frac{h}{\sqrt{\mu^2 - 1}} = \frac{12}{\sqrt{\frac{16}{9} - 1}} = \frac{36}{\sqrt{7}}$

45. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 nm]

(1) 6 m (2) 3 m (3) 5 m (4) 1 m

Sol: Ans []

46. A thin glass (refractive index 1.5) lens has optical power of 5 D in air. Its optical power in a liquid medium with refractive index 1.6 will be

(1) 25 D (2) –25 D (3) 1 D (4) –1 D

Sol: Ans []

$-5 = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (i)

$P = \left(\frac{3/2}{8/5} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (ii)

$\Rightarrow \quad -\frac{5}{P} = \frac{\frac{1}{2}}{-\frac{1}{16}} = -8$

$P = \frac{5}{8}$ There is no matching alternative

47. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are head. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?

(1) 196 Hz (2) 204 Hz (3) 200 Hz (4) 202 Hz

Sol: Ans [1]

When tape is attached to the tuning folk its frequency decreases, therefore frequency of 2, should be

$$= 200 - 4 = 196 \text{ Hz.}$$

48. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is

(1) $2\pi\alpha$ (2) $2\pi\sqrt{\alpha}$ (3) $\frac{2\pi}{\alpha}$ (4) $\frac{2\pi}{\sqrt{\alpha}}$

Sol: Ans [4]

Comparing with $\frac{d^2x}{dt^2} + \omega^2 x = 0$

$$\omega^2 = \alpha \quad \Rightarrow \quad T = \frac{2\pi}{\sqrt{\alpha}}$$

49. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

(1) remain unchanged

(2) increase towards a saturation value

(3) first increase and then decrease to the original value

(4) first decrease and then increase to the original value

Sol: Ans [3]

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is distance between point of suspension and centre of mass of the body.

As water leaks out l increase and once ball is empty centre of mass comes to initial point.

50. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?

(1) 5 % (2) 20 % (3) zero (4) 0.5 %

Sol: Ans [2]

$$f' = f\left(\frac{v - v_0}{v - v_s}\right)$$

$$f' = f\left(\frac{v + v/5}{v}\right) = \left(\frac{6f}{5}\right)$$

$$\frac{f'}{f} = \frac{6}{5} \quad \Rightarrow \quad \frac{\Delta f}{f} = \frac{1}{5}$$

$$\% \text{ change} = \frac{\Delta f}{f} \times 100 = 20\%$$

51. If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled?

(1) I_0 (2) $I_0/2$ (3) $2 I_0$ (4) $4 I_0$

Sol: Ans [1]

Intensity is independent of width of slit

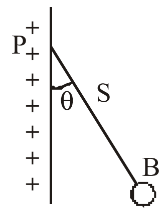
52. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is

(1) zero (2) I_0 (3) $\frac{1}{2} I_0$ (4) $\frac{1}{4} I_0$

Sol: Ans [3]

Average intensity transmitted through polarizer is $I_0/2$, where I_0 is intensity of unpolarized high.

53. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to



(1) $\sin\theta$ (2) $\tan\theta$

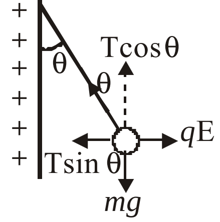
(3) $\cos\theta$ (4) $\cot\theta$

Sol: Ans [2]

For equilibrium

$T\cos\theta = mg$

$T\sin\theta = qE$



$\Rightarrow \quad \tan\theta = \frac{qE}{mg}$

$\Rightarrow \quad mg\tan\theta = q\frac{\sigma}{2\varepsilon_0}$

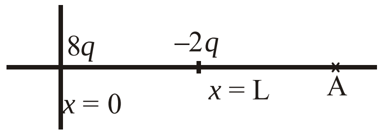
$\Rightarrow \quad \sigma \propto \tan\theta$

54. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x-axis at which the net electric field due to these two point charges is zero is

(1) 8 L (2) 4 L (3) 2 L (4) $\frac{L}{4}$

Sol: Ans [3]

Let E = 0 at point A;



$$\Rightarrow \frac{K \times 8q}{x^2} = \frac{K \times 2q}{(x - L)^2}$$

$$\pm \frac{2}{x} = \frac{1}{x - L}$$

$$\Rightarrow x = 2L, \frac{2L}{3}$$

55. Two thin rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is

(1) zero

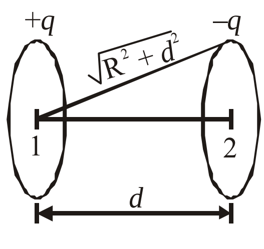
(2) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}}\right]$

(3) $\frac{Q}{4\pi\varepsilon_0 d^2}$

(4) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}}\right]$

Sol: Ans [4]

Potential at 1



$$V_1 = \frac{Kq}{R} + \frac{-Kq}{\sqrt{R^2 + d^2}}$$

$$V_2 = \frac{-Kq}{R} + \frac{Kq}{\sqrt{R^2 + d^2}}$$

$$\Rightarrow V_1 - V_2 = \frac{2Kq}{R} - \frac{2Kq}{\sqrt{R^2 + d^2}}$$

56. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C then the resultant capacitance is

(1) C (2) nC (3) $(n - 1)$C (4) $(n + 1)$C

Sol: Ans [3]

No of capacitor = (n – 1) connected in parallel

$C_{eq} = (n-1)C$

57. A fully charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference V across the capacitance is

(1) $\frac{ms\Delta T}{C}$ (2) $\sqrt{\frac{2ms\Delta T}{C}}$ (3) $\sqrt{\frac{2mC\Delta T}{s}}$ (4) $\frac{mC\Delta T}{s}$

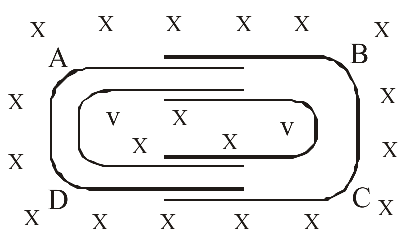
Sol: Ans [2]

Energy stored in capacitor is converted into heat by resistance, which further increases the temperature of block.

$$\Rightarrow \frac{1}{2}CV^2 = mS\Delta T$$

$$V = \sqrt{\frac{2mS\Delta T}{C}}$$

58. One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed V, then the emf induced in the circuit in terms of B, l and V where l is the width of each tube, will be



(1) zero (2) $2BlV$ (3) BlV (4) $-BlV$

Sol: Ans [2]

Induction is due to change in area.

$$\frac{d\phi}{dt} = B\frac{dA}{dt}$$

$$= B \cdot 2 \cdot l \cdot V \quad \text{(Because relative velocity = 2V)}$$

59. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be

(1) one fourth (2) halved (3) doubled (4) four times

Sol: Ans [3]

$\because$ $R \propto l$ therefore resistance will reduce to half

$\because$ $P = \dfrac{V^2}{R}$

$\Rightarrow$ Power or heat will double

60. Two thin, long, parallel wires, separated by a distance d carry a current of i A in the same direction. They will

(1) attract each other with a force of $\mu_0 i^2/(2\pi d^2)$ (2) repel each other with a force of $\mu_0 i^2/(2\pi d^2)$

(3) attract each other with a force of $\mu_0 i^2/(2\pi d)$ (4) repel each other with a force of $\mu_0 i^2/(2\pi d)$

Sol: Ans [3]

Formula (force is per unit length)

61. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be

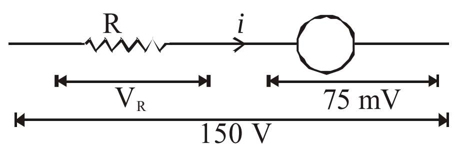
(1) 99995 (2) 9995 (3) 10^3 (4) 10^5

Sol: Ans [2]

Maximum current through galvanometer = 15 mA (150/10)

Maximum voltage across galvanometer = 75 mV(150/2)

To have 1 division/volt, Range of galvanometer = 150 V



$V_R = 150 - 0.075 = 149.925$ volt

$\Rightarrow$ $R = \dfrac{149.925}{15 \times 10^{-3}} = 9995\ \Omega$

62. Two voltameters, one of copper and another of silver, are joined in parallel. When a total charge q flows through the voltammeters, equal amount of metals are deposited. If the electrochemical equivalents of copper and silver are z_1 and z_2 respectively the charge which flows through the silver voltameter is

(1) $q\dfrac{z_1}{z_2}$ (2) $q\dfrac{z_2}{z_1}$ (3) $\dfrac{q}{1+\dfrac{z_1}{z_2}}$ (4) $\dfrac{q}{1+\dfrac{z_2}{z_1}}$

Sol: Ans [4]

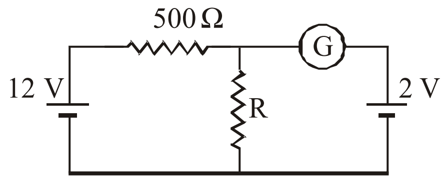
$\because$ $m = z\,q$

$z_1 q_1 = z_2 q_2$ (mass deposited is same)

$\Rightarrow$ $1 + \dfrac{q_1}{q_2} = \dfrac{z_2}{z_1} + 1$

$\Rightarrow \quad q_2 = \dfrac{z_1}{z_1 + z_2} q$

63. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be

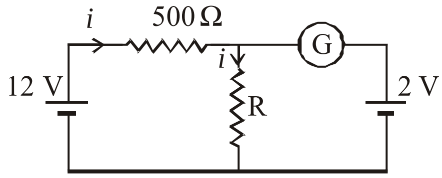


(1) 500 Ω (2) 1000 Ω

(3) 200 Ω (4) 100 Ω

Sol: Ans [4]

If there is no current through 2 V battery voltage across R = 2 volt



$i = \dfrac{12}{500 + R}$

$\Rightarrow \quad R \times \dfrac{12}{500 + R} = 2$

$\Rightarrow \quad R = 100\ \Omega$

64. Two sources of equal emf are connected to an external resistance R. The internal resistances of the two sources are R_1 and R_2 $(R_2 > R_1)$. If the potential difference across the source having internal resistance R_2 is zero, then

(1) $R = \dfrac{R_1 R_2}{(R_1 + R_2)}$ (2) $R = \dfrac{R_1 R_2}{(R_2 - R_1)}$

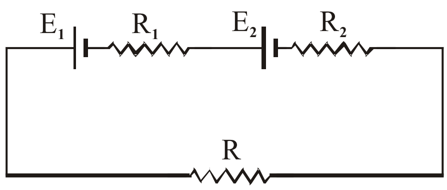
(3) $R = R_2 \times \dfrac{(R_1 + R_2)}{(R_2 - R_1)}$ (4) $R = R_2 - R_1$

Sol: Ans [4]

$\because$ Terminal voltage across $E_2 = 0$

$\Rightarrow \quad 0 = E_2 - iR_2 \quad \Rightarrow \quad i = E_2/R_2 = E/R_2$

Also, $\quad i = \dfrac{E_1 + E_2}{R_1 + R_2 + R} = \dfrac{2E}{R_1 + R_2 + R}$



$\Rightarrow \quad \dfrac{2E}{R_1 + R_2 + R} = \dfrac{E}{R_2}$

$\Rightarrow \quad R = R_2 - R_1$

65. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 ampere and 4 ampere are the currents flowing in each coil respectively. The magnetic induction in Weber/m^2 at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A·m)

(1) 5×10^{-5} (2) 7×10^{-5} (3) 12×10^{-5} (4) 10^{-5}

Sol: Ans [1]

Magnetic induction at the centre of the coil is along axis of the coil, therefore B due to each coil will be perpendicular at their common centre

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0 \times 3}{2 \times 2\pi \times 10^{-2}}\right)^2 + \left(\frac{\mu_0 \times 4}{2 \times 2\pi \times 10^{-2}}\right)^2}$$

$$= 5 \times 10^{-5}\ W/m^2$$

66. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is

(1) $\frac{2\pi q B}{m}$ (2) $\frac{2\pi m}{q B}$ (3) $\frac{2\pi mq}{B}$ (4) $\frac{2\pi q^2 B}{m}$

Sol: Ans [2]

Formula

67. In a potentiometer experiment the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is

(1) 4 Ω (2) 2 Ω (3) 1 Ω (4) 0.5 Ω

Sol: Ans [2]

$$r = R\left(\frac{l_1}{l_2} - 1\right) = 2\left(\frac{240}{120} - 1\right) = 2\,\Omega$$

68. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp when not in use?

(1) 400 Ω (2) 200 Ω (3) 40 Ω (4) 20 Ω

Sol: Ans [3]

Resistance of lamp when in use

$$R = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400\,\Omega$$

Resistance when filament is cold (i.e. lamp is not in use) = $\frac{400}{10} = 40\ \Omega$

69. A magnetic needle is kept in a non-uniform magnetic field. It experiences

(1) a force and a torque (2) a force but not a torque

(3) a torque but not a force (4) neither a force nor a torque

Sol: Ans [1]

Force on needle = $M\frac{dB}{dl}$

Where $\frac{d\mathrm{B}}{dl}$ is rate of variation of B along magnetic moment

It shows that in non uniform field force is not zero. Torque acts irrespective of nature of field.

70. A uniform electric field and a uniform magnetic field are acting the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then

(1) it will turn towards right of direction of motion

(2) it will turn towards left of direction of motion

(3) its velocity will decrease

(4) its velocity will increase

Sol: Ans [3]

Magnetic force = 0

Electric force will be opposite to E i.e., opposite to velocity in this case therefore there will be retardation.

71. A coil of inductance 300 mH and resistance 2 Ω is connected to a sources of voltage 2 V. The current reaches half of its steady state value in

(1) 0.15 s (2) 0.3 s (3) 0.05 s (4) 0.1 s

Sol: Ans [4]

$$i = i_0 \, e^{-\frac{\mathrm{R}}{\mathrm{L}}t}$$

$$\frac{1}{2} i_0 = i_0 \, e^{-\frac{2}{300\times10^{-3}}t}$$

$\Rightarrow$ $t = 0.1$ sec.

72. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of

(1) 1 μF (2) 2 μF (3) 4 μF (4) 8 μF

Sol: Ans [1]

For maximum power: $\omega = \frac{1}{\sqrt{\mathrm{LC}}}$

$\Rightarrow$ $2\pi \times 50 = \frac{1}{(10 \times \mathrm{C})^{1/2}}$

$\Rightarrow$ C = 1 μF

73. An energy source will supply a constant current into the load if its internal resistance is

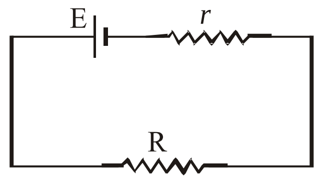
(1) zero

(2) non-zero but less than the resistance of the load

(3) equal to the resistance of the load

(4) very large as compared to the load resistance

Sol: Ans [4]



$$i = \frac{E}{R + r} = \frac{E}{r\left[1 + \frac{R}{r}\right]}$$

if $r >> R \Rightarrow i \approx \frac{E}{r}$ i.e., current is independent of load resistance.

74. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be

(1) 1.25 (2) 0.125 (3) 0.8 (4) 0.4

Sol: Ans [3]

$$\cos\phi = \frac{R}{z} = \frac{12}{15} = 0.8$$

75. The phase difference between the alternating current and emf is $\frac{\pi}{2}$. Which of the following cannot be the constituent of the circuit?

(1) L, C (2) L along (3) C alone (4) R, L

Sol: Ans [4]

For non resistance circuit; ϕ is always $\pi/2$.

AIEEE Entrance Test - 2005

Chemistry Solution

76. Which of the following oxides is amphoteric in amphoteric in character ?

(1) CaO (2) CO_2 (3) SiO_2 (4) SnO_2

Sol: Ans [4]

77. Which one of the following species is diamagnetic in nature ?

(1) He_2^+ (2) H_2 (3) H_2^+ (4) H_2^-

Sol: Ans [2]

According to M.O. theory.

78. If α is the degree of dissociation of Na_2SO_4 the vant Hoff's factor (i) used for calculating the molecular mass is

(1) $1 + \alpha$ (2) $1 - \alpha$ (3) $1 + 2\alpha$ (4) $1 - 2\alpha$

Sol: Ans [3]

$$Na_2SO_4 \rightleftharpoons 2Na^+ + SO_4$$
$$1 - \alpha \qquad 2\alpha \qquad \alpha$$

Particles after dissocaition $1- \alpha + 2a + \alpha = 1 + 2\alpha$.

79. The oxidation state of Cr in $[Cr(NH_3)_4Cl_2]^+$ is

(1) +3 (2) +2 (3) +1 (4) 0

Sol: Ans [1]

80. Hydrogen bomb is based on the principle of

(1) nuclear fission (2) natural radioactivity

(3) nuclear fusion (4) artificial radioactivity

Sol: Ans [3]

81. An ionic compound has a unit cell consisting of A ions at the corners of a cube and B ions on the centres of the faces of the cube. The empirical formula for this compound would be

(1) AB (2) A_2B (3) AB_3 (4) A_3B

Sol: Ans [3]

For $A = 8 \times \frac{1}{8} = 1$

For $B = 6 \times \frac{1}{2} = 3$ hence AB_3

82. For a spontaneous reaction, the ΔG, equilibrium constant (K) and E°_{cell} will be respectively

(1) –ve, >1, +ve (2) +ve, >1, –ve (3) –ve, <1, –ve (4) –ve, >1, –ve

Sol: Ans [1]

83. Which of the following is a polyamide?

(1) Teflon (2) Nylon-66 (3) Terylene (4) Bakelite

Sol: Ans [2]

Nylon - 66, it has $-NH-\overset{\overset{\displaystyle O}{\|}}{C}-$ group

84. Which one of the following types of drugs reduces fever ?

(1) Analgesic (2) Antipyretic (3) Antibiotic (4) Tranquiliser

Sol: Ans [2]

85. Due to the presence of an unpaired electron, free radicals are :

(1) Chemically reactive (2) Chemicaly inactive (3) Anions (4) Cations

Sol: Ans [1]

86. Lattice energy of an ionic compound depends upon

(1) Charge on the ion only (2) Size of the ion only

(3) Packing of ions only (4) Charge on the ion and size of the ion

Sol: Ans [4]

87. The highest electrical conductivity of the following aqueous solutions is of

(1) 0.1 M acetic acid (2) 0.1 M chloroacetic acid

(3) 0.1 M fluoroacetic acid (4) 0.1 M difluoroacetic acid

Sol: Ans [4]

88. Aluminium oxide may be electolysed at 1000ºC to furnish aluminium metal (at. mass = 27 amu; 1 Faraday = 96,500 Coulombs). The cathode reaction is

$Al^{3+} + 3e^- \rightarrow Al^o$

To prepare 5.12 kg of aluminium metal by this method would require

(1) 5.49×10^7 C of electricity (2) 1.83×10^7 C of electricity

(3) 5.49×10^4 C of electricity (4) 5.49×10^1 C of electricity

Sol: Ans [1]

According to Faraday's law of equivalence.

89. Consider an endothermic reaction $X \rightarrow Y$ with the activation energies E_b and E_f for the backward and forward reactions, respectively. In general

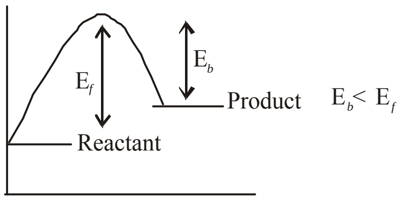
(1) $E_b < E_f$

(2) $E_b > E_f$

(3) $E_b = E_f$

(4) there is no definite relation between E_b and E_f

Sol: Ans [1]



90. Consider the reaciton : $N_2 + 3H_2 \rightarrow 2NH_3$ carried out at constant temperature and pressure. IfΔH and ΔU are the enthalpy and internal energy changes for the reaction, which of the following expressions is true ?

(1) $\Delta H = 0$ (2) $\Delta H = \Delta U$ (3) $\Delta H < \Delta U$ (4) $\Delta H > \Delta U$

Sol: Ans [3]

$\Delta H = \Delta U + \Delta n gRT = 0 - 2RT$

Hence $\Delta H < \Delta U$

91. Which one of the following statements is NOT true about the effect of an increase in temperature on the distribution of molecular speeds in a gas ?

(1) The most probable speed increases

(2) The fraction of the molecules with the most probable speed increases

(3) The distribution becomes broader

(4) The area under the distribution curve remains the same as under the lower temperature

Sol: Ans [2]

92. The volume of a colloidal particel, V_C as compared to the volume of a solute particle in a true solution V_S, could be

(1) $\frac{V_C}{V_S} \approx 1$ (2) $\frac{V_C}{V_S} \approx 10^{23}$ (3) $\frac{V_C}{V_S} \approx 10^{-3}$ (4) $\frac{V_C}{V_S} \approx 10^{3}$

Sol: Ans [4]

$$\frac{V_C}{V_S} \approx \frac{10^{-4}}{10^{-7}} \approx 10^3$$

93. The solubility product of a salt having general formula MX_2, in water is : 4×10^{-12}. The concentration of M^{2+} ions in the aqueous solution of the salt is

(1) 2.0×10^{-6} M (2) 1.0×10^{-4} M (3) 1.6×10^{-4} M (4) 4.0×10^{-10} M

Sol: Ans [2]

$K_{SP} = 4S^3$; $S = 1.0 \times 10^{-4}$ M

94. Benzene and toluene form nearly ideal solutions. At 20ºC, thevapour pressure of benzene is 75 torr and that of toluene is 22 torr. The partial vapour pressure of benzene at 20ºC for a solution containing 78 g of benzene and 46 g of toluene in torr is

(1) 50 (2) 25 (3) 37.5 (4) 53.5

Sol: Ans [1]

According to Raoult's law, partial pressure of benzene = $p^o.X$

95. The exothermic formation of ClF_3 is represented by the equation :

$$Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)}; \ \Delta rH = -329 \text{ kJ}$$

Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3?

(1) Increasing the temperature (2) Removing Cl_2

(3) Increasing the volume of the container (4) Adding F_2

Sol: Ans [4]

According to Le-Chattelllier's principle

96. Two solutions of a substance (non electrolyte) are mixed in the following manner. 480 ml of 1.5 M first solution + 520 mL of 1.2 M second solution. What is the molarity of the final mixture ?

(1) 1.20 M (2) 1.50 M (3) 1.344 M (4) 2.70 M

Sol: Ans [3]

Applying $M_1V_1 + M_2V_2 = M_3V_3$

$M_3 = 1.344$ M

97. For the reaction

$$2NO_{2(g)} \rightleftharpoons 2NO_{(g)} + O_{2(g)}$$

$(K_C = 1.8 \times 10^{-6}$ at 184ºC)

(R = 0.0831 kJ / (mol.K))

When K_P and K_C are compared at 184ºC it is foudn that

(1) K_P is greater than K_C

(2) K_P is less than K_C

(3) $K_P = K_C$

(4) Whether K_P is greater than, less than or equal to K_C depends upon the total gas pressure

Sol: Ans [1]

Applying $K_P = K_C(RT)^{\Delta n}$, $K_P = K_C(RT) = K_C = \frac{K_P}{RT}$

98. Hydrogen ion concentration in mol/L in a solution of pH = 5.4 will be :

(1) 3.98×10^{8} (2) 3.88×10^{6} (3) 3.68×10^{-6} (4) 3.98×10^{-6}

Sol: Ans [4]

pH = 5.4 $H^+ = 10^{-5.4} = 3.98 \times 10^{-6}$

99. A reacion involving two different reactants can never be

(1) unimolecular reaction (2) first order reaction

(3) second order reaction (4) bimolecular reaction

Sol: Ans [1]

100. If we consider that 1/6, in place of 1/12, mass of carbon atom is taken to be the relative atomic mass unit, the mass of one mole of substance will

(1) decrease twice

(2) increase two fold

(3) remain unchanged

(4) be a function of the molecular mass of the substance

Sol: Ans [2]

101. In a multi-electron atom, which of the following orbitals described by the three quantum number will will have the same energy in the absence of magnetic and elctric fields ?

(a) $n = 1, l = 0, m = 0$ (b) $n = 2, l = 0, m = 0$

(c) $n = 2, l = 1, m = 1$ (d) $n = 3, l = 2, m = 0$

(e) $n = 3, l = 2, m = 0$

(1) (a) and (b) (2) (b) and (c) (3) (c) and (d) (4) (d) and (e)

Sol: Ans [4]

102. During the process of electrolytic refining of copper, some metals present as impurity settle as 'anode mud'. These are

(1) Sn and Ag (2) Pb and Zn (3) Ag and Au (4) Fe and Ni

Sol: Ans [3]

103. .

Electrolyte	KCl	KNO3	HCl	NaOAc	NaCl
Λ^{∞}(S cm^2 mol^{-1})	149.9	145.0	426.2	91.0	126.5

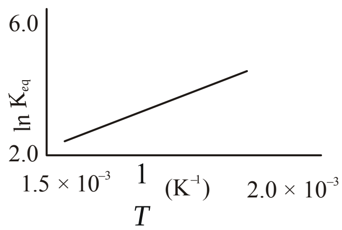
Calculate Δ^{∞}_{HOAc} using appropriate molar conductances of the electrolytes listed above at infinite dilution in H_2O at 25ºC

(1) 517.2 (2) 552.7 (3) 390.7 (4) 217.5

Sol: Ans [3]

Applying Kohlrausch's law

104. A schematic plot of lnK_{eq} versus inverse of temperature for a reaction is shown below



The reaction must be

(1) exothermic (2) endothermic

(3) one with negligible enthalpy change (4) highly spontaneous at ordinary temperature

Sol: Ans [1]

This is applied for Exothermic reaction

105. The disperse phase in colloidal iron (III) hydroxide and colloidal gold is positively and negatively charged, respectivel. Which of the following statements is NOT correct?

(1) Magnesium chloride solution coagulates, the gold sol more readily than the iron (III) hydroxide sol.

(2) Sodium sulphate solution causes coagulation in both sols

(3) Mixing the sols has no effect

(4) Coagulation in both sols can be brought about by electrophoresis

Sol: Ans [3]

106. Based on lattice energy and other considerations which one of the following alkali metal chlorides is expected to have the highest melting point ?

(1) LiCl (2) NaCl (3) KCl (4) RbCl

Sol: Ans [1]

107. Heating mixture of Cu_2O and Cu_2S will give

(1) $Cu + SO_2$ (2) $Cu + SO_3$ (3) $CuO + CuS$ (4) Cu_2SO_3

Sol: Ans [1]

$$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$$

108. The molecular shapes of SF_4, CF_4 and XeF_4 are

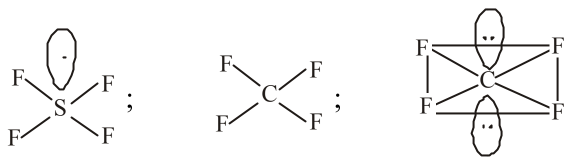
(1) the same with 2, 0 and 1 lone pairs of electrons

(2) the same with 1, 1 and 1 lone pair of electrons on the central atoms, respectively

(3) different with 0, 1 and 2 lone pairs of electrons on the central atom, respectively

(4) different with 1, 0 and 2 lone pairs of electrons on the central atom, respectively

Sol: Ans [4]



109. The number and type of bonds between two carbon atoms in calcium carbide are

(1) One sigma, one pi (2) One sigma, two pi (3) Two sigma, one pi (4) Two sigma, two pi

Sol: Ans [2]

$^{\ominus}C \equiv C^{\ominus}$

110. The oxidation state of chromium in the final product formed by the reaction between Kl and acidified potassium dichromate solution is

(1) +4 (2) +6 (3) +2 (4) +3

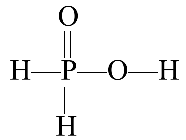
Sol: Ans [4]

Acidic $K_2Cr_2O_7$ acts as oxidision agent itself reduced from Cr(+6) to Cr(+3).

111. The number of hydrogen atom(s) attached to phosphorus atom in hypophosphorous acid is

(1) zero (2) two (3) one (4) three

Sol: Ans [2]



112. What is the conjugate base of OH^- ?

(1) O_2 (2) H_2O (3) O^- (4) O^{2-}

Sol: Ans [4]

O^{2-} ; $OH^- \xrightarrow{-H^+} O^{2-}$

113. The correct order of the thermal stability of hydrogen halides (H – X) is

(1) $HI > HBr > HCl > HF$ (2) $HF > HCl > HBr > HI$

(3) $HCl < HF > HBr < HI$ (4) $HI > HCl < HF > HBr$

Sol: Ans [2]

114. Heating an aqueous solution of aluminium chloride to dryness will give

(1) $AlCl_3$ (2) Al_2Cl_6 (3) Al_2O_3 (4) $Al(OH)Cl_2$

Sol: Ans [3]

115. Calomel (Hg_2Cl_2) on reaction with ammonium hydroxide gives

(1) $HgNH_2Cl$ (2) $NH_2–Hg–Hg–Cl$ (3) Hg_2O (4) HgO

Sol: Ans [1]

116. In which of the following arrangements the order is NOT according to the property indicated against it ?

(1) $Al^{3+} < Mg^{2+} < Na^+ < F^-$:
Increaing ionic size

(2) $B < C < N < O$:
Increasing first ionization enthalpy

(3) $I < Br < F < Cl$:
Increasing electron gain enthalpy
(with negative sign)

(4) $Li < Na < K < Rb$:
Increasing metallic radius

Sol: Ans [2]

117. In silicon dioxide

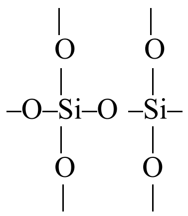
(1) each silicon atom is surrounded by four oxygen atoms and each oxygen atom is bonded to two silicon atoms

(2) each silicon atom is surrounded by two oxygen atoms and each oxygen atom is bonded to two silicon atoms

(3) silicon atom is bonded to two oxygen atoms

(4) there are double bonds between silicon and oxygen atoms

Sol: Ans [1]



118. Of the following sets which one does NOT contain isoelectronic species ?

(1) PO_4^{3-}, SO_4^{2-}, Cl_4^- (2) CN–, N2, C_2^{2-},

(3) SO_3^{2-}, CO_3^{2-}, NO_3^- (4) BO_3^{3-}, CO_3^{3-}, NO_3^-

Sol: Ans [3]

Odd atom S belongs to third period.

119. The lanthanide contration is responsible for the fact that

(1) Zr and Y have about the same radius (2) Zr and Nb have similar oxidation state

(3) Zr and Hf have about the same radius (4) Zr and Zn have the same oxidation state

Sol: Ans [3]

Zr and Hf have about the same radius

120. The IUPAC name of the coordination compound $K_3[Fe(CN)_6]$ is

(1) Potassium hexacyanoferrate (II) (2) Potassium hexacyanoferrate (III)

(3) Potassium hexacyanoiron (II) (4) Tripotassium hexacyanoiron (II)

Sol: Ans [2]

121. Which of the following compounds shows optical isomerism ?

(1) $[Cu(NH_3)_4]^{2+}$ (2) $[ZnCl_4]^{2-}$ (3) $[Cr(C_2O_4)_3]^{3-}$ (4) $[Co(CN)_6]^{3-}$

Sol: Ans [3]

122. Which one of the following cyano complexes would exhibit the lowest value of paramagnetic behaviour?

(1) $[Cr(CN)_6]^{3-}$ (2) $[Mn(CN)_6]^{3-}$ (3) $[Fe(CN)_6]^{3-}$ (4) $Co(CN)_6]^{3-}$

(At. Nos : Cr = 24, Mn = 25, Fe = 26, Co = 27)

Sol: Ans [4]

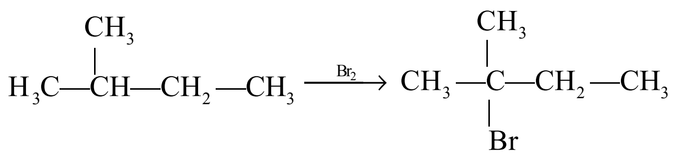
$[Co(CN)_6]^{3-}$; central metal atom Co has no unpaired electrons.

123. 2-Methylbutane on reacting with bromine in the presence of sunlight gives mainly

(1) 1-bromo-2-methylbutane (2) 2-bromo-2-methylbutane

(3) 2-bromo-3-methylbutane (4) 1-bromo-3-methylbutane

Sol: Ans [2]



124. A photon of hard gamma radiation knocks a proton out of $^{24}_{12}Mg$ nucleus to form

(1) the isotope of parent nucleus
(2) the isobear of parent nucleus
(3) the nuclide $^{23}_{11}Na$
(4) the isobar of $^{23}_{11}Na$

Sol: Ans [3]

$$^{24}_{12}\text{Mg} \longrightarrow ^{23}_{11}\text{Na} + ^{1}_{1}\text{P}$$

125. The best reagent to convert pent-3-en-2-ol into pent-3-en-2-one is

(1) Acidic permanganate
(2) Acidic dichromate
(3) Chromic anhydride in glacial acetic acid
(4) Pyridinium chloro-chromate

Sol: Ans [4]

126. Tertiary alkyl halides are practically inert to substitution by S_{N_2} mechanism because of

(1) insolubility
(2) instability
(3) inductive effect
(4) steric hindrace

Sol: Ans [4]

127. In both DNA and RNA, heterocylic base and phosphate ester linkages are at

(1) C'_5 and C'_2 respectively of the sugar molecule
(2) $C'_2$and C'_5 respectively of the sugar molecule
(3) C'_1 and C'_5 respectively of the sugar molecule
(4) C'_5 and C'_1 respectively of the sugar molecule

Sol: Ans [3]

128. Reaction of one molecule of HBr with one molecule of 1, 3-butadiene at 40ºC gives predominantly

(1) 3-bromobutene under kinetically controlled conditions
(2) 1-bromo-2-butene under thermodynamically controlled conditions
(3) 3-bromobutene under thermodynamically controlled conditions
(4) 1-bromo-2-butene under kinetically controlled conditions

Sol: Ans [2]

$$CH_2 = CH - CH = CH_2 \xrightarrow[1,4-addition]{HBr} Br\ CH_2{-}CH = CH - CH_3$$

129. Among the following acids which has the lowest pK $_a$ value ?

(1) CH_3COOH
(2) HCOOH
(3) $(CH_3)_2CH - COOH$
(4) CH_3CH_2COOH

Sol: Ans [2]

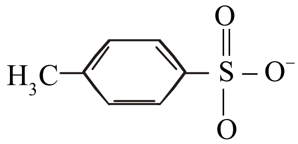
$H-\overset{\overset{O}{||}}{C}-OH$ is strongest among the given acids hence lowest pK_a value.

130. The decreasing order of nucleophilicity among the nucleophiles

(a) $CH_3\underset{\underset{O}{||}}{C}-O^-$

(b) CH_3O^-

(c) CN^-

(d) 

(1) (a), (b), (c), (d) (2) (d), (c), (b), (a) (3) (b), (c), (a), (d) (4) (c), (b), (a), (d)

Sol: Ans [3]

131. Which one of the following methods is neither meant for the synthesis nor for separation of amines ?

(1) Hinsberg method (2) Hofmann method (3) Wurtz reaction (4) Curtius reaction

Sol: Ans [3]

Wutz reaction is used for synthesis of alkanes.

132. Which of the following is fully fluorinated polymer ?

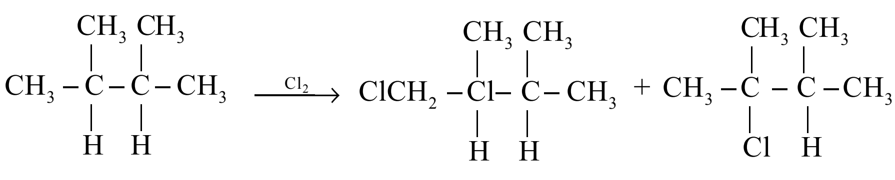
(1) Neoprene (2) Teflon (3) Thiokol (4) PVC

Sol: Ans [2]

133. Of the five isomeric hexanes, the isomer which can give two monochlorinated compounds is

(1) n-hexane (2) 2, 3-dimethylbutane (3) 2, 2-dimethylbutane (4) 2-methylpentane

Sol: Ans [2]



134. Alkyl halides react with dialkyl copper reagents to give

(1) alkenes (2) alkyl copper halides (3) alkanes (4) alkenyl halides

Sol: Ans [3]

$$2\,R-X+R'_2Cu \rightarrow 2R-R'+CuX_2$$

135. Acid catalyzed hydration of alkenes except ethene leads to the formation of

(1) primary alcohol

(2) secondary or tertiary alcohol

(3) mixture of primary and secondary alcohols

(4) mixture of secondary and tertiary alcohols

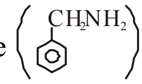
Sol: Ans [2]

Except ethene, other alkenes on acid catalysed hydration form 2º or 3º alcohols, due to the formation of more stable carbocation

136. Amongest the following the most basic compound is

(1) benzylamine (2) aniline (3) acetanilide (4) p-nitroaniline

Sol: Ans [1]

In benzyl amine  the lone pair of electons is not involved in resonance with phenyl ring as in all other cases.

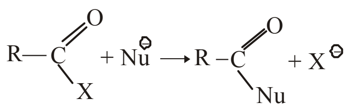
137. Which types of isomerism is shown by 2, 3-dichlorobutane ?

(1) Diastereo (2) Optical (3) Geometric (4) Sturctural

Sol: Ans [2]

The given compound has 2 chiral centres and forms 3 stereoisomers d, l and meso.

138. The reaction



is fastest when X is

(1) Cl (2) NH_2 (3) OC_2H_5 (4) OCOR

Sol: Ans [1]

–Cl being weakest base is a good leaving group.

139. Elimination of bromine from 2-bromobutane results in the formaiton of

(1) equimolar mixture of 1 and 2-butene (2) predominantly 2-butene

(3) prdominantly 1-butene (4) predominantly 2-butyne

Sol: Ans [2]

Saytzeff rule

140. Equimolar solutions in the same solvent have

(1) Same boiling point but different freezing point (2) Same freezing point but different boiling point

(3) Same boiling and same freezing points (4) Different boiling and different freezing points

Sol: Ans [3]

Factual, based on the fact that colligative properties depend upon the number of particles

141. Which of the following statements in relation to the hydrogen atom is correct ?

(1) $3s$ orbital is lower in energy than $3p$ orbital

(2) $3p$ orbital is lower in energy than $3d$ orbital

(3) $3s$ and $3p$ orbitals are of lower energy than $3d$ orbital

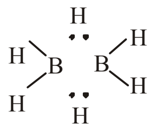
(4) $3s$, $3p$ and $3d$ orbitals all have the same energy

Sol: Ans [1]

142. The structure of diborane (B_2H_6) contains

(1) four 2c-2e bonds and two 3c-2e bonds
(2) two 2c-2e bonds and four 3c-2e bonds
(3) two 2c-2e bonds and two 3c-3e bonds
(4) four 2c-2e bonds and four 3c-2e bonds

Sol: Ans [1]



Diborane has 4 two centre-two electron bonds and 2 three centre-two electron bonds.

143. The value of the 'spin only' magnetic moment for one of the following configurations is 2.84 BM. The correct one is

(1) d^4 (in strong ligand field)
(2) d^4 (in weak ligand field)
(3) d^3 (in weak as well as in strong fields)
(4) d^5 (in strong ligand field)

Sol: Ans [1]

According to CFT, splitting between t_{2g} and e_g groups is large in strong field and fourth electron gets paired with 3rd electron corresponding to 2 unpaired electrons.

144. Which of the following factors may be regarded as the main cause of lanthanide contraction ?

(1) Poor shielding of one of 4f electron by another in the subshell
(2) Effective shielding of one of 4f electrons by another in the subshell
(3) Poorer shielding of 5d electrons by 4f electrons
(4) Greater shielding of 5d electron by 4f electrons

Sol: Ans [3]

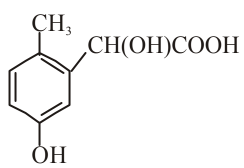
f orbitals have poor schielding effect.

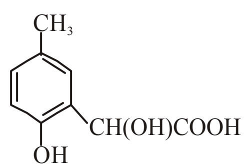
145. Reaction of cyclohexanone with dimethylamine in the presence of catalytic amount of an acid forms a compound if water during the reaction is continuously removed. The compound formed is generally known as

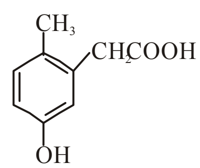
(1) a Schiff's base (2) an enamine (3) an imine (4) an amine

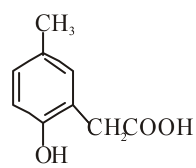
Sol: Ans [2]

146. p-cresol reacts with chloroform in alkaline medium to give the compound A which adds hydrogen cyanide to form, the compound B. The latter on acidic hydrolysis gives chiral carboxylic acid. The structure of the carboxylic acid is

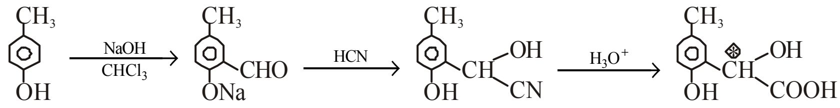
(1) 

(2) 

(3) 

(4) 

Sol: Ans [2]



⊛ Chiral carbon

147. An organic compound having molecular mass 60 is found to contain C = 20%, H = 6.67% heating it gives NH_3 alongwith a solid residue. the solid residue give violet colour with alkaline copper sulphate solution. The compound is

(1) CH_3NCO (2) CH_3CONH_2 (3) $(NH_2)_2CO$ (4) $CH_3CH_2CONH_2$

Sol: Ans [3]

Ammonia satisfies all the aforesaid data and also the emperical formula is CH_4N_2O

148. If the bond dissociation energies of XY, X_2 and Y_2 (all diatomic molecules) are in the ratio of 1 : 1 : 0.5 and $\Delta_f H$ for the formation of XY is –200 kJ mole $^{-1}$. The bond dissociation energy of X_2 will be

(1) 100 kJ mol^{-1} (2) 200 kJ mol^{-1} (3) 300 kJ mol^{-1} (4) 400 kJ mol^{-1}

Sol: Ans [2]

XY and X_2 have dissociation energies in the ratio 1 : 1 hence heat of formation of XY = dissociation enregy of X_2 (with opp. sign).

149. $t_{\frac{1}{4}}$ can be taken as the time taken for the concentration of a reactant to drop to $\frac{3}{4}$ of its initial value. If the rate constant for a first order reaction is K, the $t_{\frac{1}{4}}$ can be written as

(1) 0.10 / K (2) 0.29 / K (3) 0.69 / K (4) 0.75 / K

Sol: Ans [2]

$$Kt_{1/4} = 2.303 \ \log \frac{1}{314}$$

150. An amount of solid NH_4HS is placed in a flask already containing ammonia gas at a certain temperature and 0.50 atm. pressure. Ammonium hydrogen sulphide decoomposes to yield NH_3 and H_2S gases in the flask. When the decomposition reaction reaches equilibrium, the total pressure in the flask rises to 0.84 atm. The equilibrium constant for NH_4HS decomposition at this temperature is

(1) 0.30 (2) 0.18 (3) 0.17 (4) 0.11

Sol: Ans [4]

$$NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$$

At equilibirum $\quad 1 - P \quad\quad 0.5 + P \quad\quad P$

Total pressure at equilibrium

$0.5 + P + P = 0.84$

or $P = 0.17$

$K_P = (0.5 + P)\, P = 0.17 \times 0.67 = 0.11$

AIEEE 2005

MATHEMATICS

1. The differential equation representing the family of curves $y^2 = 2c\left(x+\sqrt{C}\right)$, where $C > 0$, is a parameter, is of order and degree as follows :

(1) order 1, degree 3 (2) order 2, degree 2 (3) order 1, degree 2 (4) order 1, degree 1

Ans. (1)

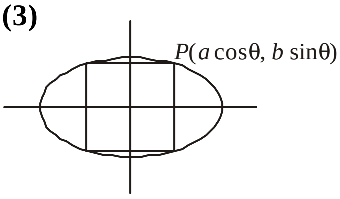
Sol.: The given equation is differentiated once and c is eliminated which gives $y^2 + 4x^2y'^2 - 4xyy' = 4yy'^3$. Hence the order is 1 and degree is 3.

2. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(1) $\sqrt{ab}$ (2) $\frac{a}{b}$ (3) $2ab$ (4) ab

Ans. (3)

Sol.:



From the figure, it is clear that area of the rectangle $= 4ab \cos\theta \sin\theta$

$= 2ab \sin 2\theta$

Hence, maximum area $= 2ab$.

3. $\lim\limits_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+\cdots+\frac{1}{n}\sec^2 1\right]$ equals

(1) tan 1 (2) $\frac{1}{2}\tan 1$ (3) $\frac{1}{2}\sec 1$ (4) $\frac{1}{2}\operatorname{cosec} 1$

Ans. (2)

Sol.: The general term $T_r = \frac{r}{n^2}\sec^2\left(\frac{r}{n}\right)^2$. Put $\frac{r}{n}=x$ and $\frac{1}{n}=dx$ which gives the value of limit

$$\int_0^1 x\sec^2 x^2\,dx = \frac{\tan 1}{2}$$

4. If the cube root of unity are 1, w, w^2 then roots of equation $(x-1)^3 + 8 = 0$, are

(1) $-1, 1-2w, 1-2w^2$ (2) $-1, 1+2w, 1+2w^2$

(3) $-1, -1+2w, -1-2w^2$ (4) $-1, -1, -1$

Ans. (1)

Sol.: The given equation is $(x-1)^3 + 8 = 0$

This implies $\left(\frac{x-1}{2}\right)^3 = -1 \Rightarrow \left(\frac{x-1}{2}\right) = -1, -\omega$ and $-\omega^2$

$\Rightarrow \quad x = -1,\ 1 - 2\omega$ and $1 - 2\omega^2$

5. If $A^2 - A + I = 0$, then the inverse of A is

(1) $A - I$ (2) $I - A$ (3) $A + I$ (4) A

Ans. (2)

Sol.: Multiplying the given equation by A^{-1}, we get $A - I + A^{-1} = 0 \Rightarrow A^{-1} = I - A$.

6. Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is

(1) an equivalence relation (2) reflexive and symmetric only

(3) reflexive and transitive only (4) reflexive only

Ans. (3)

Sol.: Since (3, 3), (6, 6), (9, 9), (12, 12) are the members of $R \Rightarrow R$ is reflexive.

Again, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \Rightarrow R$ is transitive.

7. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

(1) 25.5 (2) 24.0 (3) 22.0 (4) 20.5

Ans. (3)

Sol.: $\text{mode} = \frac{\text{mean} + 2\text{median}}{3} = \frac{21 + 2\times 22}{3} = 21.66 \approx 22$.

8. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(1) $x^2 + 4y + 2 = 0$ (2) $x^2 - 4y + 2 = 0$ (3) $y^2 - 4x + 2 = 0$ (4) $y^2 + 4x + 2 = 0$

Ans. (3)

Sol.: Let the point Q be $(2t^2, 4t)$ and mid point of PQ be (h, k) then $h = \frac{1 + 2t^2}{2}$ and $k = \frac{0 + 4t}{2}$

Eliminating t, we get $y^2 - 4x + 2 = 0$.

9. If C is the mid point of AB and P is any point outside AB, then

(1) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$ (2) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$ (3) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$ (4) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

Ans. (3)

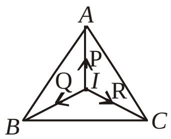
Sol.: The position vector of mid point of AB is $\overrightarrow{PC} = \frac{\overrightarrow{PA} + \overrightarrow{PB}}{2}$

10. *ABC* is a triangle. Forces **P**, **Q**, **R** acting along *IA*, *IB* and *IC* respectively are in equilibrium, where *I* is the incentre of ΔABC. Then **P** : **Q** : **R** is

(1) $\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$ (2) $\cos A : \cos B : \cos C$

(3) $\sin A : \sin B : \sin C$ (4) $\sin\frac{A}{2}:\sin\frac{B}{2}:\sin\frac{C}{2}$

Ans. (1)

Sol.: 

$$\angle BIC = \pi - \left(\frac{B+C}{2}\right), \angle AIB = \pi - \left(\frac{A+B}{2}\right), \angle AIC = \pi - \left(\frac{A+C}{2}\right)$$

Applying Lami's theorem, we get, $\dfrac{P}{\sin\frac{B+C}{2}} = \dfrac{Q}{\sin\frac{A+C}{2}} = \dfrac{R}{\sin\frac{A+B}{2}}$

$$\Rightarrow \frac{P}{\cos\frac{A}{2}} = \frac{Q}{\cos\frac{B}{2}} = \frac{C}{\cos\frac{C}{2}}$$

11. In a triangle *PQR*, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then

(1) $b = c$ (2) $b = a + c$ (3) $a = b + c$ (4) $c = a + b$

Ans. (4)

Sol.: $\tan\frac{P}{2} + \tan\frac{Q}{2} = -\frac{b}{a}$ and $\tan\frac{P}{2}\tan\frac{Q}{2} = \frac{C}{a}$, $\tan\left(\frac{P+Q}{2}\right) = 1$ gives $a + b = c$.

12. If the coefficient of *r*th, $(r + 1)$th and $(r + 1)$th and $(r + 2)$th terms in the binomial expansion of $(1 + y)^m$ are in A.P., then *m* and *r* satisfy the equation

(1) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$ (2) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

(3) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$ (4) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$

Ans. (1)

Sol.: Since ${}^mC_{r-1}, {}^mC_r, {}^mC_{r+1}$ are in A.P. $\Rightarrow$ $2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$

which gives $m^2 - m(4r+1) + 4r^2 - 2 = 0$

13. Let f : (–1, 1) $\rightarrow$ B, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is interval

(1) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (3) $\left(0, \frac{\pi}{2}\right)$ (4) $\left[0, \frac{\pi}{2}\right)$

Ans. (2)

Sol.: Since $\tan^{-1}\frac{2x}{1-x^2}$ is an increasing function so the range B will be $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

14. If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation

(1) $\frac{a}{b} = 1$ (2) $ab = 1$ (3) $a - b = 1$ (4) $a + b = 1$

Ans. (1)

Sol.: The coefficient of x^7 in the expansion of $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11} = {}^{11}C_5\, a^6\, b^{-5}$ and coefficient of x^{-7} in the expansion of $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11} = {}^{11}C_6\, a^5\, b^{-6}$. On equating, we get $a/b = 1$.

15. If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1$, then z lies on

(1) a straight line (2) a parabola (3) an ellipse (4) a circle

Ans. (1)

Sol.: $\omega = \frac{z}{z - \frac{1}{3}i} \Rightarrow \frac{|z|}{\left|z - \frac{i}{3}\right|} = 1 \Rightarrow |z| = \left|z - \frac{i}{3}\right|$ which is bisector of the line joining origin and the point (i/3).

16. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1 + a^2 x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2 x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2 x \end{vmatrix}$, then $f(x)$ is a polynomial of degree

(1) 3 (2) 2 (3) 1 (4) 0

Ans. (2)

Sol.: $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 1 & x+b^2x & x+c^2x \\ 1 & 1+b^2x & x+c^2x \\ 1 & x+b^2x & 1+c^2x \end{vmatrix} = (1-x)^2$$

17. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg z_1 – arg z_2 is equal to

(1) 0 (2) $\frac{-\pi}{2}$ (3) $\frac{\pi}{2}$ (4) $-\pi$

Ans. (1)

Sol.: Given that $|z_1 + z_2| = |z_1| + |z_2|$ which is possible when Arg z_1 = Arg z_2 $\Rightarrow$ Arg z_1 – Arg z_2 = 0.

18. The value of a for which the sum of the squares of the roots of the equation

$x^2 - (a-2)x - a - 1 = 0$ assume the least value is

(1) 3 (2) 2 (3) 1 (4) 0

Ans. (3)

Sol.: Let α and β be roots of equation, then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a-1)^2 + 5$ which gives $a = 1$.

19. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals

(1) 2 (2) 1 (3) –2 (4) 3

Ans. (2)

Sol.: The difference of roots = 1 $\Rightarrow$ $b^2 - 4c = 1$.

20. The system of equations

$\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

has no solution, if α is

(1) not –2 (2) 1 (3) –2 (4) either –2 or 1

Ans. (3)

Sol.: The value of the coefficient determinent $D = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$ which gives $(\alpha - 1)^2 [\alpha + 2] = 0$. So,

$\alpha = 1$ or $\alpha = -2$ but for $\alpha = 1$ there are infinite number of solutions of the given system.

21. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(1) ${}^{56}C_3$ (2) ${}^{56}C_4$ (3) ${}^{55}C_4$ (4) ${}^{55}C_3$

Ans. (2)

Sol.: ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3 = {}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3.$

$= {}^{50}C_3 + {}^{50}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 = {}^{56}C_4.$

22. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction

(1) $A^n = nA + (n-1)I$ (2) $A^n = 2^{n-1}A + (n-1)I$

(3) $A^n = nA - (n-1)I$ (4) $A^n = 2^{n-1}A - (n-1)I$

Ans. (3)

Sol.: $A = I + \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$. The two matrices on the right commute, hence by the Binomial theorem

$A^n = I + n\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$ because powers higher than or equal to 2 of the matrix $\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$ are 0.

23. If the letters of word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(1) 603 (2) 602 (3) 601 (4) 600

Ans. (3)

Sol.: The alphabatical order of letters is A, C, H, I, N, S. The number of words which begin with any of the five letters A, C, H, I, N are 120 × 5 = 600. The next word will be SACHIN so its rank. is 601.

24. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in

(1) Arithmetic – Geometric Progression (2) HP

(3) GP (4) AP

Ans. (2)

Sol.: $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n \Rightarrow x = \frac{1}{1-a}$, $y = \frac{1}{1-b}$ and $z = \frac{1}{1-c}$

Since a, b, c are in A.P. x, y, z are in A.P.

25. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

(1) $-\dfrac{3}{8}x^2$ (2) $\dfrac{x}{2}-\dfrac{3}{8}x^2$ (3) $1-\dfrac{3}{8}x^2$ (4) $3x+\dfrac{3}{8}x^2$

Ans. (1)

Sol.: Upto terms of order x^2

$$\frac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}=\left[1+\frac{3}{2}x+\frac{3}{8}x^2-1-\frac{3}{2}x-\frac{3}{4}x^2\right]\left[1+\frac{1}{2}x+\frac{3}{8}x^2\right]=-\frac{3}{8}x^2$$

26. If $\cos^{-1}x-\cos^{-1}\dfrac{y}{2}=\alpha$, then $4x^2-4xy\cos\alpha+y^2$ is equal to

(1) $4\sin^2\alpha$ (2) $-4\sin^2\alpha$ (3) $2\sin 2\alpha$ (4) 4

Ans. (1)

Sol.: Given that $\cos^{-1}x-\cos^{-1}\dfrac{y}{2}=\alpha$, $\Rightarrow$ $\cos(\cos^{-1}x-\cos^{-1}y/2)=\cos\alpha$

$\Rightarrow$ $\dfrac{xy}{2}+\sqrt{1-x^2}\sqrt{1-y^2/4}=\cos\alpha$ $\Rightarrow$ $4x^2+y^2-4xy\cos\alpha=4\sin^2\alpha$

27. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then $\sin A$, $\sin B$, $\sin C$ are in

(1) Arithmetic – Geometric Progression (2) H.P.

(3) G.P. (4) A.P.

Ans. (4)

Sol.: The altitudes of the triangle are $\dfrac{2\Delta}{a},\dfrac{2\Delta}{b},\dfrac{2\Delta}{c}$ are given in H.P.

$\Rightarrow$ $\sin A$, $\sin B$, $\sin C$ are in A.P.

28. In a triangle ABC, let $\angle C=\dfrac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2(r+R)$ equals

(1) $a+b+c$ (2) $c+a$ (3) $b+c$ (4) $a+b$

Ans. (4)

Sol.: For a right angled triangle $R=c/2$ and $r=\dfrac{ab}{a+b+c}$ $\Rightarrow$ $2(r+R)=(a+b)$

29. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

(1) $\left(-\infty, \frac{1}{3}\right]$ (2) $(-\infty, -4]$ (3) $(-\infty, \infty)$ (4) $[2, \infty)$

Ans. (1)

Sol.: Given that $f(x) = 3x^2 - 2x + 1 \Rightarrow f'(x) = 6x - 2 \geq 0 \Rightarrow x \geq \frac{1}{3}$

30. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim\limits_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to

(1) $\frac{-a^2}{2}(\alpha - \beta)^2$ (2) $\frac{1}{2}(\alpha - \beta)^2$ (3) $\frac{a^2}{2}(\alpha - \beta)^2$ (4) 0

Ans. (3)

Sol.: $\lim\limits_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2} = \lim\limits_{x \to \alpha} \frac{1 - \cos a(x - \alpha)(x - \beta)}{(x - \alpha)^2} = \frac{a^2}{2}(\alpha - \beta)^2$ because $\lim\limits_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$.

31. The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta + \theta\cos\theta)$ at any point 'θ' is such that

(1) it passes through $\left(a\frac{\pi}{2}, -a\right)$ (2) it is at a constant distance from the origin

(3) it passes through the origin (4) it makes angle $\frac{\pi}{2} + \theta$ with the x-axis

Ans. (2)

Sol.: The equation of normal to the curve at any point is $y \sin \theta + x \cos \theta = a$ which is at a constant distance from origin.

32. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then

(1) $f(6) < 5$ (2) $f(6) = 5$ (3) $f(6) \geq 8$ (4) $f(6) < 8$

Ans. (3)

Sol.: Applying Lagrange's mean value theorem, $\frac{f(6) - f(1)}{6 - 1} = f'(c) \ \forall c \in (1,6)$

$\Rightarrow \frac{f(6) + 2}{5} \geq 2 \Rightarrow f(6) \geq 8$

33. If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \le (x - y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

(1) 2 (2) 1 (3) –1 (4) 0

Ans. (4)

Sol.: $|f(x) - f(y)| \le (x - y)^2$, $x, y \in R$ taking the limit as $x \to y$, we get $|f'(x)| \le 0 \Rightarrow f'(x) = 0 \Rightarrow f(x)$ is a constant function $\Rightarrow f(1) = 0$

34. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0} \frac{1}{h} f(1+h) = 5$, then $f'(1)$ equals

(1) 5 (2) 6 (3) 3 (4) 4

Ans. (1)

Sol.: Since $\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$ exists and $\lim_{h\to 0} \frac{1}{h} f(1+h)$ exists, it follows that $\lim_{h\to 0} \frac{f(1)}{h}$ exists

Hence, $f(1) = 0$ and $f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{f(1+h)}{h} = 5$.

35. $\int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$ is equal to

(1) $\frac{xe^x}{1+x^2} + C$ (2) $\frac{x}{(\log x)^2 + 1} + C$ (3) $\frac{\log x}{(\log x)^2 + 1} + C$ (4) $\frac{x}{x^2 + 1} + C$

Ans. (2)

Sol.: Let $I = \int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$. Put $\log x = t$, then $I = \int e^t \left(\frac{1}{(1+t^2)} - \frac{2t}{(1+t^2)^2} \right) dt$

$$= \frac{e^t}{1+t^2} + C = \frac{x}{1 + (\log x)^2} + C$$

36. A spherical ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

(1) $\frac{1}{54\pi}$ cm/min. (2) $\frac{5}{6\pi}$ cm/min. (3) $\frac{5}{36\pi}$ cm/min. (4) $\frac{1}{18\pi}$ cm/min.

Ans. (4)

Sol.: Let $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow 50 = 4\pi(225)\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{18\pi}$ cm/m

37. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x=\frac{\pi}{4}$ and $x=\beta>\frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\,\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is

(1) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$ (2) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$ (3) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (4) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

Ans. (2)

Sol.: Let $A=\int_{\pi/4}^{\beta} f(x)\,dx=\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\,\beta$. Hence $\frac{dA}{d\beta}=f(\beta)=\sin\beta+\beta\cos\beta-\frac{\pi}{4}\sin\beta+\sqrt{2}$

$\Rightarrow\ f\left(\frac{\pi}{2}\right)=1-\frac{\pi}{4}+\sqrt{2}$

38. Let $f: R \to R$ be a differentiable function having $f(2) = 6$, $f'(2)=\left(\frac{1}{48}\right)$. Then $\lim\limits_{x\to 2}\int_{6}^{f(x)}\frac{4t^3}{x-2}\,dt$ equals

(1) 12 (2) 18 (3) 24 (4) 36

Ans. (2)

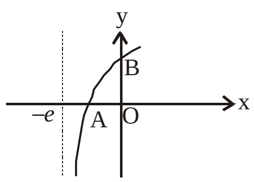
Sol.: By L'Hospital's rule $\lim\limits_{x\to 2}\int_{6}^{f(x)}\frac{4t^3}{x-2}\,dt=\lim\limits_{x\to 2} f'(x)\,4[f(x)]^3=\frac{4}{48}\times 216=18$

39. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is

(1) 3 (2) 4 (3) 1 (4) 2

Ans. (3)

Sol.:



Required area = Area of the sector OAB =

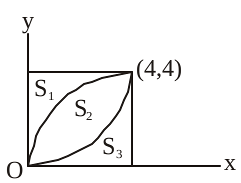
$\int_{1-e}^{0}\ln(x+e)\,dx=\int_{1}^{e}\ln u\,du=[u\ \ln u\ -u]_1^e=1$

40. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

(1) 2 : 1 : 2 (2) 1 : 1 : 1 (3) 1 : 2 : 1 (4) 1 : 2 : 3

Ans. (2)

Sol.:



$S_3=\int_0^4\frac{x^2}{4}\,dx=\frac{64}{12}$, $S_1=\int_0^4\frac{y^2}{4}\,dy=\frac{64}{12}$, $S_2=16-S_1-S_3=16-\frac{128}{12}=\frac{64}{12}$

41. If $I_1 = \int_0^1 2^{x^2}\,dx$, $I_2 = \int_0^1 2^{x^3}\,dx$, $I_3 = \int_1^2 2^{x^2}\,dx$, and $I_4 = \int_1^2 2^{x^3}\,dx$ then

(1) $I_3 = I_4$ (2) $I_3 > I_4$ (3) $I_2 > I_1$ (4) $I_1 > I_2$

Ans. (4)

Sol.: $2^{x^2} > 2^{x^3}$ if $0 < x < 1$. Hence $I_1 > I_2$.

42. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(1) above the x-axis at a distance of $\frac{3}{2}$ from it (2) above the x-axis at a distance of $\frac{2}{3}$ from it

(3) below the x-axis at a distance of $\frac{3}{2}$ from it (4) below the x-axis at a distance of $\frac{2}{3}$ from it

Ans. (3)

Sol.: The equation of the required line can be written as $ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$. The coefficient of x must be zero which implies λ equals $-a/b$. Substituting this value the equation of the line is $y = -3/2$.

43. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

(1) $\log\left(\frac{y}{x}\right) = cx$ (2) $\log\left(\frac{x}{y}\right) = cy$ (3) $y\log\left(\frac{x}{y}\right) = cy$ (4) $x\log\left(\frac{y}{x}\right) = cy$

Ans. (1)

Sol.: The given equation can be written as $\frac{dy}{dx} = \frac{y}{x}\left(\ln\frac{y}{x}\right) + 1$. Substituting $u = \frac{y}{x}$ gives the differential equation $x\frac{du}{dx} = u\ln u$ whose solution is $\ln u = cx$.

44. If a vertex of a triangle is (1, 1) and the mid points of two sides through the vertex are (–1, 2) and (3, 2), then the centriod of the triangle is

(1) $\left(1, \frac{7}{3}\right)$ (2) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (3) $\left(-1, \frac{7}{3}\right)$ (4) $\left(\frac{-1}{3}, \frac{7}{3}\right)$

Ans. (1)

Sol.: The coordinates of the other two vertices are obtained using section formula and are (–3, 3) and (5, 3). Hence the centroids has coordinates (1, 7/3).

45. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for

(1) infinitely many values of a (2) exactly two values of a

(3) exactly one value of a (4) no value of a

Ans. (4)

Sol.: The equatin of the common chord is $5ax + (c - d)y + a + 1 = 0$. This should be the same as $5x + by - a = 0$. Hence $\frac{5a}{5} = \frac{c - d}{b} = \frac{a+1}{-a}$ which gives the quadratic $a^2 + a + 1 = 0$ which has no real roots.

46. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point. That point is

(1) (1, –2) (2) $\left(1, -\frac{1}{2}\right)$ (3) (–1, 2) (4) (–1, –2)

Ans. (1)

Sol.: $\frac{1}{a} + \frac{1}{c} - \frac{2}{b} = 0$. Comparing this with the equation $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ the required point is (1, –2)

47. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is

(1) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 \;\; p^2) = 0$ (2) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

(3) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$ (4) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

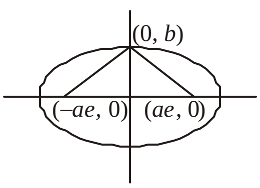
Ans. (2)

Sol.: Let the centre be (h, k) and suppose the equation of the circle is $x^2 + y^2 - 2hx - 2ky + c = 0$. The orthogonality condition gives $c = p^2$. Substituting this value of c and putting $x = a, y = b$ in the equation of the circle gives the locus $2ax + 2by - (a^2 + b^2 + p^2) = 0$.

48. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

(1) $\frac{1}{4}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

Ans. (3)

Sol.: 

By Pythagoras theorem $2(b^2 + a^2e^2) = 4a^2e^2$. Hence $b^2 / a^2 = e^2$. For an ellipse $b^2/a^2 = 1 - e^2$ which gives $e = 1/\sqrt{2}$.

49. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is

(1) a hyperbola (2) a parabola (3) an ellipse (4) a circle

Ans. (2)

Sol.: A circle which touches the x axis has equation $x^2 + y^2 + 2gx + 2fy + g^2 = 0$. This touches the given circle if $g^2 + f^2 + 6f + 9 = (f \pm 2)^2$. Hence the locus is parabola.

50. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(1) 45º (2) 30º (3) 0º (4) 90º

Ans. (4)

Sol.: The direction ratios of the two lines are $\left(\frac{1}{2}, \frac{1}{3}, -1\right)$ and $\left(\frac{1}{6}, -1, -\frac{1}{4}\right)$. The dot product of these two vectors is zero. Hence the angle is 90º.

51. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ the value of λ is

(1) $\frac{3}{4}$ (2) $\frac{-4}{3}$ (3) $\frac{5}{3}$ (4) $\frac{-3}{5}$

Ans. (3)

Sol.: The directin ratio of the line is $\mathbf{e} = (1, 2, 2)$. The direction ratio of the normal to the plane is $\left(2, -1, \sqrt{\lambda}\right)$

$\mathbf{n} \cdot \mathbf{e} = 3\sqrt{5+\lambda}\sin\theta$ gives $2\sqrt{\lambda} = \sqrt{5+\lambda}$. So $\lambda = \frac{5}{3}$

52. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(1) a parabola (2) a hyperbola (3) an ellipse (4) a circle

Ans. (2)

Sol.: Condition for tangency gives $\beta^2 = a^2\alpha^2 - b^2$. Hence the locus is the hyperbola.

53. The distance between the line $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ and the plane $\mathbf{r}.(\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = 5$ is

(1) $\frac{3}{10}$ (2) $\frac{10}{3}$ (3) $\frac{10}{9}$ (4) $\frac{10}{3\sqrt{3}}$

Ans. (4)

Sol.: The distance can be obtained by taking any point on the line and any point on the plane and projecting the vector joining these two points along the unit normal to the plane. A point on the line is $2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and the point on the plane is $\mathbf{j}$. The unit normal is $\frac{\mathbf{i} + 5\mathbf{j} + \mathbf{k}}{3\sqrt{3}}$. The required distance is $\frac{10}{3\sqrt{3}}$.

54. For any vector **a**, the value of $(\mathbf{a} \times \mathbf{i})^2 + (\mathbf{a} \times \mathbf{j})^2 + (\mathbf{a} \times \mathbf{k})^2$ is equal to

(1) $2\mathbf{a}^2$ (2) $4\mathbf{a}^2$ (3) $3\mathbf{a}^2$ (4) $\mathbf{a}^2$

Ans. (1)

Sol.: Use $|\mathbf{a} \times \mathbf{i}|^2 = |\mathbf{a}|^2 - (\mathbf{a} \cdot \mathbf{i})^2$ etc. gives $2|\mathbf{a}|^2$.

55. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals

(1) –2 (2) 2 (3) –1 (4) 1

Ans. (1)

Sol.: The centres of the two spheres are (–3, 4, 1) and (5, –2, 1) whose mid point is (1, 1, 1). This lies on the plane. We get $a = -2$.

56. Let a, b and c be distinct non-negative numbers. If the vectors $a\mathbf{i} + a\mathbf{j} + c\mathbf{k}$, $\mathbf{i} + \mathbf{k}$ and $c\mathbf{i} + c\mathbf{j} + b\mathbf{k}$ lie in a plane, then c is

(1) equal to zero (2) the Harmonic Mean of a and b

(3) the Geometric Mean a and b (4) the Arithmetic Mean of a and b

Ans. (3)

Sol.: The scalar triple product of the three vectors must be zero which gives $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

implying $ab = c^2$

57. If **a**, **b**, **c** are non-coplanar vectors and λ is a real number then $\left[\lambda(\mathbf{a}+\mathbf{b})\ \lambda^2\mathbf{b}\ \lambda\mathbf{c}\right] = \left[\mathbf{a}\ \mathbf{b}+\mathbf{c}\ \mathbf{b}\right]$ for

(1) exactly three values of λ (2) exactly two values of λ

(3) exactly one value of λ (4) no value of λ

Ans. (4)

Sol.: $\left[\lambda(\mathbf{a}+\mathbf{b})\ \lambda^2\mathbf{b}\ \lambda\mathbf{c}\right] = \lambda^4[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$, $\left[\mathbf{a}\ \ \mathbf{b}+\mathbf{c}\ \ \mathbf{b}\right] = -[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$. Hence the equation has no solution for λ.

58. Let $\mathbf{a} = \mathbf{i} - \mathbf{k}$, $\mathbf{b} = x\mathbf{i} + \mathbf{j} + (1 - x)\mathbf{k}$ and $\mathbf{c} = y\mathbf{i} + x\mathbf{j} + (1 + x - y)\mathbf{k}$. Then [**a**, **b**, **c**] depends on

(1) both x and y (2) neither x nor y (3) only y (4) only x

Ans. (4)

Sol.: $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix} = 1 + 2x$

59. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(1) $\frac{8}{9}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{1}{9}$

Ans. (4)

Sol.: Number of points in the sample space is 27 and the number of points favourable to the event is 3. Hence, the required probability is $\frac{1}{9}$.

60. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals

(1) $1-\frac{3}{e^2}$ (2) $\frac{3}{e^2}$ (3) $\frac{2}{e^2}$ (4) 0

Ans. (1)

Sol.: $P(X > 1.5) = 1 - P(X = 0) - P(X = 1) = 1-\frac{3}{e^2}$

61. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, whree $\overline{A}$ stands for component of event A. Then events A and B are

(1) independent but not equally likely
(2) mutually exclusive and independent
(3) equally likely and mutually exclusive
(4) equally likely but not independent

Ans. (1)

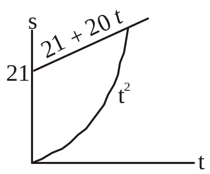
Sol.: $P(A) = \frac{3}{4}, P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{4} \Rightarrow P(B) = \frac{1}{3}$. So, $P(A \cap B) = P(A)P(B)$.

62. A lizard, at initial distance of 21 cm behind an insect, moves from rest with an acceleration of 2 cm/s^2 and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard catch the insect after

(1) 21 s (2) 24 s (3) 20 s (4) 1 s

Ans. (1)

Sol.:



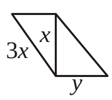
$t^2 = 21 + 20t$ gives t = 21 s.

63. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is

(1) 3 : 2 (2) $3:2\sqrt{2}$ (3) 2 : 1 (4) $3:\sqrt{2}$

Ans. (2)

Sol.:



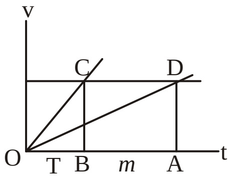
By Pythagoras theorem, $9x^2 = x^2 + y^2$. So, $\frac{3x}{y} = \frac{3}{\sqrt{8}}$

64. Two points A and B move from rest along a straight line with constant acceleration f and f' respectively. If A takes m sec. more than B and describes 'n' units more than B in acquiring the same speed then

(1) $\frac{1}{2}(f + f')m = ff'n^2$ (2) $(f' - f)n = \frac{1}{2}ff'm^2$

(3) $(f - f')m^2 = ff'n$ (4) $(f + f')m^2 = ff'n$

Ans. (2)

Sol.: 

Area of triangle OAD – Area of triangle $OCB = n$ gives $n = \frac{1}{2} f'T$

$f(m+T) = f'T \Rightarrow (f' - f)T = fm$. Using the first equation, we get,

$2n(f' - f) = ff'm^2$

65. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with the. The resultant of A and B after combining is displaced through a distance

(1) $\frac{H}{2(A+B)}$ (2) $\frac{H}{A-B}$ (3) $\frac{2H}{A-B}$ (4) $\frac{H}{A+B}$

Ans. (4)

Sol.: When the forces A and B are acting on the plane, the distance of resultant from the force A is equal to $\frac{dB}{A+B}$ where d is the distance between the points of action of the forces A and B. When the couple of moment H is also acting on the plane, then distance of the resultant force from the force $A = \frac{dB}{A+B} - \frac{H}{A+B}$. Thus the position of resultant is shifted by $\frac{H}{A+B}$.

66. The sum of the series : $1 + \frac{1}{4.2!} + \frac{1}{16.4!} + \frac{1}{64.4!} + \dots$ ad inf. is

(1) $\frac{e-1}{2\sqrt{e}}$ (2) $\frac{e+1}{2\sqrt{e}}$ (3) $\frac{e-1}{\sqrt{e}}$ (4) $\frac{e+1}{\sqrt{e}}$

Ans. (2)

Sol.: General term = $\frac{1}{2^n\ n!}, n = 0, 2, 4$

$$\sum_{n\text{ even}} \frac{x^n}{n!} = \frac{e^x + e^{-x}}{2}$$

67. If $a_1, a_2, a_3, ..., a_n$ are in G.P., then the determinant $\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to

(1) 4 (2) 2 (3) 1 (4) 0

Ans. (4)

Sol.: $\log a_n, \log a_{n+1}, \log a_{n+2}$ are in A.P.

68. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

(1) $(-\infty, 4)$ (2) $[4, 5]$ (3) $(5, 6]$ (4) $(6, \infty)$

Ans. (1)

Sol.: Discreminent equals $-4(k - 5) \geq 0 \Rightarrow k \leq 5$. The quadratic equation at $x = 5$ must be +ve and sum of the roots must be less than 10. These conditions implie $k^2 - 9k + 20 > 0$. So, $k < 4$.

69. If the equation $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has a positive root, which is

(1) greater than or equal to α (2) equal to α

(3) greater than α (4) smaller than α

Ans. (4)

Sol.: The expression $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1$ is the derivative of $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x$. So, by Rolle's theorem, the derivative is 0 at some point between 0 and α.

70. A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$, where a is a given constant and $f(0) = 1$, $f(2a - x)$ is equal to

(1) $f(a) + f(a - x)$ (2) $f(-x)$ (3) $-f(x)$ (4) $f(x)$

Ans. (3)

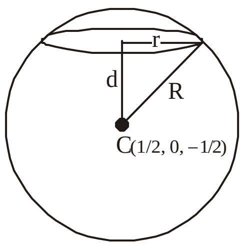
Sol.: Put $y = 0$ in the given functional equation to get $f(a) = 0$. Next put $x = 0$ to get $f(-y) = f(y)$. Next, put $x = y = a$ to get $f(2a) = -1$. Finally, put $x = 2a$ and replace y by x to get $f(2a - x) = -f(x)$.

71. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius

(1) 2 (2) $\sqrt{2}$ (3) 3 (4) 1

Ans. (4)

Sol.:



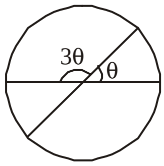
$$r^2 = R^2 - d^2 = \frac{5}{2} - \frac{9}{6} = 1$$

72. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then

(1) $3a^2 + 10ab + 3b^2 = 0$ (2) $3a^2 + 2ab + 3b^2 = 0$

(3) $3a^2 - 10ab + 3b^2 = 0$ (4) $3a^2 - 2ab + 3b^2 = 0$

Ans. (2)

Sol.: 

From the diagram it is clear that θ equals 45º,

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\left[(a+b)^2 - ab\right]^{1/2}}{a+b} \text{ gives } 3a^2 + 2ab + 3b^2 = 0$$

73. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx,\ a > 0,$ is

(1) $\frac{\pi}{a}$ (2) 2π (3) $a\pi$ (4) $\frac{\pi}{2}$

Ans. (4)

Sol.: $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx = \int_{-\pi}^{\pi} \frac{\cos^2(\pi - \pi - x)}{1+a^{\pi-\pi-x}}\,dx$ gives $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx = \frac{1}{2}\int_{-\pi}^{\pi} \cos^2 x\,dx = \frac{\pi}{2}$.

74. A particle is projected from a point O with velocity u at an angle of 60º with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by

(1) $\frac{2u}{3}$ (2) $\frac{u}{\sqrt{3}}$ (3) $\frac{u}{3}$ (4) $\frac{u}{2}$

Ans. (2)

Sol.: $\mathbf{v} = \mathbf{u} + \mathbf{g}\,t$ $\mathbf{v}\cdot\mathbf{u} = 0$ gives $t = \frac{2|\mathbf{u}|}{\sqrt{3}\,g}$. This implies $|\mathbf{v}| = \frac{|\mathbf{u}|}{\sqrt{3}}$

75. Let $x_1, x_2, ..., x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is

(1) 9 (2) 12 (3) 15 (4) 18

Ans. (4)

Sol.: By Cauchy-Schwarz inequality, $\left|\sum_{i=1}^{n} x_i\right| \le \sqrt{n}\left(\sum x_i^2\right)^{\frac{1}{2}}$ we get $n \ge 16$.